Differential and Integral Equations, Volume 4, Number 3, May 1991, pp. 449–457.

QUASILINEAR ELLIPTIC PROBLEMS WITH MEASURES AS DATA

Jean Michel Rakotoson
Département de Mathématiques, Université de Poitiers
40, Avenue du recteur Pineau, 86022 Poitiers, France

(Submitted by: Roger Temam)

Introduction. This paper is a continuation of earlier works [8-11] concerning quasilinear problems involving measures as data; particularly we are interested in the existence of solutions for the equations (and also to the corresponding inequalities):

Find $u \in W_0^{1,q}(\Omega)$, *satisfying* $Au = \mu \in M(\Omega)$ (*set of bounded Radon measures*).

Recent results in this direction have been obtained in [1], [2], [4] (see also references therein); the main differences with our results are:

The method that we use is new and different, especially to get priori estimates (see Lemma 1) and compactness results (see Lemma 2).

The operator that we use satisfies the general Leray-Lions conditions (see [6]); in particular it might depend on the solution u and its gradient and it is just strictly monotonic.

Lemma 2 gives a simpler use than Lemma 2.2 in [7, p. 184] (see also [6]) and we will show in a later work that this lemma leads to a simplification for the proof of existence in many quasilinear problems. For, the sake of completness, we give an example in the last paragraph.

The organization of our paper will be:

1. Assumptions on the operator A;
2. Two fundamental lemmas;
3. The case of the equations;
4. The corresponding inequalities;
5. An example of an application of compactness results.

1. Assumptions on A. Let Ω be a bounded open set of $\mathbb{R}^N$. We denote $Au = -\operatorname{div}(\hat{\mathbf{a}}(x,u,Du))$, where $\hat{\mathbf{a}}$ is a Caratheodory vector valued function from $\Omega \times \mathbb{R} \times \mathbb{R}^N$ into $\mathbb{R}^N$ having the following standard Leray-Lions properties:

(H1) There exist $c > 0$, $k \in L^{p'}(\Omega)$ $(\frac{1}{p} + \frac{1}{p'} = 1)$, $\alpha > 0$, $2 - (1/N) < p \leq N$, such that

$$\begin{array}{ll} \text{(growth)} & |\hat{\mathbf{a}}(x,u,\xi)| \leq c(|u|^{p-1} + |\xi|^{p-1} + k(x)), \\ \text{(monotonicity)} & [\hat{\mathbf{a}}(x,u,\xi) - \hat{\mathbf{a}}(x,u,\zeta)][\xi - \zeta] > 0, \quad \xi \neq \zeta, \\ \text{(coercivity)} & \hat{\mathbf{a}}(x,u,\xi)\cdot\xi \geq \alpha|\xi|^p. \end{array}$$

Received for publication December 3, 1990.
AMS Subject Classifications: 35J85, 35J60, 35J65.