Differential and Integral Equations, Volume 3, Number 4, July 1990, pp. 757–766.

ON SIEVED ORTHOGONAL POLYNOMIALS VI: DIFFERENTIAL EQUATIONS

Joaquin Bustoz
Department of Mathematics, Arizona State University, Tempe, Arizona 85287 USA

Mourad E.H. Ismail†
Department of Mathematics, University of South Florida, Tampa, Florida 33620 USA

Jet Wimp‡
Department of Mathematics, Drexel University, Philadelphia, PA 19104 USA

(Submitted by: F.V. Atkinson)

Abstract. We show how to derive a second order differential equation with polynomial coefficients from differential recurrence relations (ladder operators) with polynomial coefficients. We then find the ladder operators for the sieved ultraspherical polynomials of the second kind and use them to derive a differential equation for the latter polynomials.

1. Introduction. Al-Salam, Allaway, and Askey [1] introduced a very interesting family of orthogonal polynomials, called the sieved ultraspherical polynomials. The sieved ultraspherical polynomials of the first and second kinds are orthogonal on $[-1,1]$ with respect to the weight functions $w_1(x)$, and $w_2(x)$; respectively

$$w_1(x) = |\sin(k\theta)|^{2\lambda}/\sin\theta, \quad w_2(x) = \sin\theta|\sin(k\theta)|^{2\lambda}, \quad x = \cos\theta. \tag{1.1}$$

Here k is a given positive integer. Proofs of the orthogonality of the sieved ultraspherical polynomials with respect to the aforementioned weight functions are in [2] and [3]. Al-Salam, Allaway, and Askey [1] raised the question of finding a differential equation satisfied by the sieved ultraspherical polynomials. One reason for the interest in finding such a differential equation is that all the points where the weight functions vanish will be singularities of the differential equations satisfied by the polynomials. If all these singularities are regular then we will have a concrete example of a differential equation satisfied by known orthogonal polynomials which has as many regular singular points as one pleases.

The purpose of this paper is to construct a differential equation satisfied by the sieved ultraspherical polynomials of the second kind. It turned out that the

Received January 9, 1989.
†Partially supported by NSF grant DMS 8714630.
‡Partially supported by NSF grant DMS 8802381.
AMS Subject Classifications: 34A05, 33A65, 42C05.