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INVESTIGATION OF A MATHEMATICAL MODEL OF THERMOELASTICITY

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(Submitted by: Jerome A. Goldstein)

1. Introduction. We consider the boundary value problem

$$
\begin{aligned}
&\partial\theta/\partial t - \kappa\Delta\theta + \kappa\eta \cdot \partial(\operatorname{div}\vec{v})/\partial t = -Q,\\
&\partial^2\vec{v}/\partial t^2 - \rho^{-1}[\mu\Delta\vec{v} + (\lambda+\mu)\operatorname{grad}\operatorname{div}\vec{v}] + \rho^{-1}\gamma\cdot\operatorname{grad}\theta = \vec{F}\\
&\qquad (0 \le t \le T,\ x \in \Omega);\\
&\theta = 0, \quad \vec{v} = \vec{0} \quad (0 \le t \le T,\ x \in \partial\Omega);\\
&\theta(0,x) = \theta^0(x), \quad \vec{v}(0,x) = \vec{v}^{\,0}(x),\\
&\vec{v}_t^{\,\prime}(0,x) = \vec{v}^{\,1}(x) \quad (x \in \overline{\Omega})
\end{aligned}
\tag{1}
$$

which describes the motion and the temperature change of a homogeneous isotropic elastic medium occupying the volume Ω with density ρ, Lame's constants λ, μ and thermal coefficients γ, κ, η. Here θ is a deviation of temperature, $\vec{v} = (v_1, v_2, v_3)$ is a vector of elastic displacements, Q is an outward source of heat, $\vec{F} = (F_1, F_2, F_3)$ is a vector of volume density of outward forces, Ω is an open bounded domain in $\mathbb{R}^3$ with boundary $\partial\Omega \in C^2$, and $\overline{\Omega} = \Omega \cup \partial\Omega$ (see [1]).

Problem (1) is reduced to the Cauchy problem for a system of two differential equations of parabolic and hyperbolic types in Banach space. In this way the existence and uniqueness theorem of solutions of problem (1) was obtained under conditions on the data which are close to the necessary conditions.

Earlier the existence and uniqueness theorem was obtained by Galerkin's method in [2, 3]. However, this result was obtained under more restrictions on the data of the problem. I think that this is connected with the fact that better properties of a parabolic operator in comparison with a hyperbolic one are not used.

2. The formulation of the result. The choice of the functional spaces in which the solution of problem (1) is sought is connected with the type of the system under investigation. It is natural to investigate a multidimensional hyperbolic equation in

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