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THE FUNCTIONAL SPACE $C^{-1,\alpha}$ AND ANALYTIC SEMIGROUPS

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Abstract. The functional space $C^{-1,\alpha}$ of derivatives of Hölder continuous functions is introduced. Using integral estimates, it is also proved that a variational elliptic operator generates an analytic semigroup in $C^{-1,\alpha}$. After characterizing the interpolation and extrapolation spaces, some applications to linear and quasilinear parabolic equations are given.

1. Introduction. The main result of this paper is the proof of generation of analytic semigroups by variational elliptic operators with Dirichlet boundary conditions in the space $(C^{-1,\alpha}(\Omega))^N$, $(0<\alpha<1)$, where Ω is any bounded domain in $\mathbf{R}^N$ with sufficiently smooth boundary $\partial\Omega$, and $(C^{-1,\alpha}(\Omega))^N$ (in the sequel we will omit the index N when there is no danger of confusion) is the set of all N-uples of functions belonging to $C^{-1,\alpha}(\Omega)$. The space $C^{-1,\alpha}(\Omega)$ is the set of all functions $u\in H^{-1}(\Omega)$ (the dual space of $H_0^1(\Omega)$) having a α-Hölder continuous representation: i.e., there are $f_0,\dots,f_n\in C^\alpha(\bar\Omega)$, such that $u=f_0+\sum_{i=1}^n D_i f_i$ in the distributional sense. $C^{-1,\alpha}(\Omega)$ is a Banach space with the norm

$$\|u\|_{-1,\alpha}=\inf\Big\{\sum_{i=0}^n\|f_i\|_{0,\alpha}\,;\ f_i\in C^\alpha(\bar\Omega),\ u=f_0+\sum_{i=1}^n D_i f_i\Big\}. \tag{1.1}$$

If $A_{i,j},\ A_i:\bar\Omega\to C^{N^2}$ $(i,j=1,\dots,n)$ are α-Hölder continuous, satisfy the ellipticity condition

$$\sum_{i,j=1}^n \eta_i\eta_j\ \mathrm{Re}\,\langle A_{ij}^{hk}\pi_h,\pi_k\rangle\ge\nu|\eta|^2|\pi|^2\quad\text{for each }\eta\in\mathbf{R}^n\text{ and }\pi\in\mathbf{C}^N,$$

with $\nu>0$ and B_i and $C:\Omega\to\mathbf{C}^{N^2}$ are bounded and measurable, we define the operator

$$\begin{cases} Eu=-\sum_{i,j=1}^n D_i(A_{ij}D_ju)-\sum_{i=1}^n D_i(A_iu)+\sum_{i=1}^n B_iD_iu+Cu\\[2ex] E:D(E)=\{u\in C^{1,\alpha}(\bar\Omega)\cap C_0^0(\bar\Omega)\}\to C^{-1,\alpha}(\Omega),\end{cases}\tag{1.2}$$

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