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MULTIPLE PERIODIC SOLUTIONS FOR SOME NONLINEAR ORDINARY DIFFERENTIAL EQUATIONS OF HIGHER ORDER

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Abstract. Let $m \geq 3$ be an integer and $g(t, u)$ a function defined in $\mathbf{R}^2$, 2π-periodic in t and satisfying the coercivity condition

$$g(t,u) \to +\infty \quad \text{as} \quad |u| \to +\infty$$

uniformly in t. We consider the periodic boundary value problem of order m

$$\begin{cases} \pm\, u^{(m)} + g(t,u) = s \\ u^{(i)}(0) = u^{(i)}(2\pi), \quad i = 0, 1, \ldots, m-1, \end{cases} \tag{P}$$

and we study additional conditions on g under which the following theorem holds: there exist real numbers $s_0 \leq s_1$ such that (P) has zero, at least one or at least two solutions according to whether $s < s_0$, $s = s_1$ or $s > s_1$.

1. Introduction and auxiliary lemmas. Let $m \geq 3$ be an integer and $g(t, u)$ a function defined in $\mathbf{R}^2$, 2π-periodic in t. We denote by L the m-th order ordinary differential operator defined by

$$Lu = \epsilon u^{(m)}$$

where $\epsilon = \pm 1$. Consider the periodic boundary value problem

$$\begin{cases} Lu + g(t,u) = s \\ u^{(i)}(0) = u^{(i)}(2\pi), \quad i = 0, 1, \ldots, m-1. \end{cases} \tag{P}$$

where $s \in \mathbf{R}$ is a parameter and the following coerciveness condition is assumed throughout.

$$\lim_{|u| \to +\infty} g(t,u) = +\infty \quad \text{uniformly in} \quad t \in [0, 2\pi]. \tag{H}$$

Moreover, g is supposed to be a Carathéodory function (in some instances continuous). For our purpose it is convenient to introduce the following hypothesis.

$$\begin{cases} \text{Given } R > 0 \text{ there exists a function } \phi \in L^2(0, 2\pi) \text{ such that} \\ |g(t,u)| \leq \phi(t) \text{ for a.e. } t \in [0, 2\pi] \text{ and all } u \in [-R, R]. \end{cases} \tag{H'}$$

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