Differential and Integral Equations, Volume 2, Number 1, January 1989, pp. 63-79.

ON THE SOLVABILITY OF A SEMILINEAR TWO-POINT BVP AROUND THE FIRST EIGENVALUE

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(Submitted by : J. Mawhin)

Abstract. Existence and multiplicity results for semilinear two-point BVPs are proved, under suitable assumptions on the interaction of the nonlinear term with the first eigenvalue of the associated homogeneous linear problem. New nonresonance conditions are introduced.

1. Introduction. This work is devoted to the solvability of the nonlinear two-point boundary value problem

$$-u'' = g(u) + h(x) \qquad (' = d/dx) \tag{1.1}$$

$$u(a) = r_1, \quad u(b) = r_2 \tag{1.2}$$

where it is assumed throughout the paper that $g : \mathbb{R} \to \mathbb{R}$ is a continuous function, $h : [a, b] \to \mathbb{R}$ is Lebesgue integrable and r_1, $r_2 \in \mathbb{R}$ are fixed constants. Solutions to (1.1) are intended in the Carathéodory sense.

As is well known, (1.1)-(1.2) is a special case of the semilinear elliptic problem with Dirichlet boundary conditions

$$-\Delta u = g(u) + h(x) \quad \text{in } \Omega, \tag{1.3}$$

$$u = \phi \quad \text{in } \partial\Omega, \tag{1.4}$$

where Ω is a bounded open subset of $\mathbb{R}^N$, with a smooth boundary $\partial\Omega$, and ϕ is a given function.

Received for publication March 3, 1988.

†On leave of absence from Faculdade de Ciências da Universidade de Lisboa, with a scholarship of Fundação Calouste Gulbenkian.

‡Work performed under the auspicies of G.N.A.F.A. - C.N.R. and partially supported by the fund M.P.I. 60%, ZAN 6Q5.

AMS Subject Classifications: 34B15.