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EIGENFUNCTION EXPANSIONS ASSOCIATED WITH A MULTIPARAMETER SYSTEM OF DIFFERENTIAL EQUATIONS

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Abstract. In this paper we resolve a difficulty arising in the partial differential equation approach to the eigenfunction expansion problem associated with a multiparameter system of ordinary differential equations of the second order and then establish new results concerning the uniform convergence of the eigenfunction expansion for a left definite eigenvalue problem.

1. Introduction. In the partial differential equation approach to the eigenfunction expansion problem associated with a multiparameter system of ordinary differential equations, a difficulty arises which does not appear to have been explicitly dealt with in the literature. Here, we resolve this difficulty and then establish new results concerning the uniform convergence of the eigenfunction expansion associated with the so-called left definite eigenvalue problem [19]. The methods presented here can also be applied to similar problems in which the left definite assumption is no longer valid, as for example, in those multiparameter problems considered in [9] and [10]. We would remark that this is important from the point of view of future applications; we shall return to this particular aspect later.

Accordingly, we shall be concerned here with the simultaneous n-parameter systems ($n \geq 2$),

$$(p_r(x_r)y_r')' + \left(\sum_{s=1}^{n} \lambda_s A_{rs}(x_r) - q_r(x_r)\right) y_r = 0, \quad 0 \leq x_r \leq 1, \quad ' = d/dx_r, \tag{1.1.r}$$

$$y_r(0)\cos\alpha_r - p_r(0)y_r'(0)\sin\alpha_r = 0, \quad 0 \leq \alpha_r \leq \pi/2, \tag{1.2.r}$$

$$y_r(1)\cos\beta_r - p_r(1)y_r'(1)\sin\beta_r = 0, \quad \pi/2 \leq \beta_r \leq \pi, \tag{1.3.r}$$

for $r = 1, \dots, n$, where it is supposed that in $0 \leq x_r \leq 1$ ($r = 1, \dots, n$), $p_r > 0$ and of class $C^{2n^*+1,1}$, the A_{rs} are real and of class $C^{2n^*,1}$, while $q_r \geq 0$ and of class $C^{2n^*,1}$, where $n = 4n^* + k$, $0 \leq k \leq 3$. We also suppose that if $\alpha_r = \beta_r = \pi/2$ for $r = 1, \dots, n$, then the q_r are not all identically zero. Writing x for $(x_1, \dots, x_n)$, I^n for the product of the

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