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A RANDOM SCHROEDINGER EQUATION: WHITE NOISE MODEL

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Abstract. This paper is essentially an application of the author's theory of abstract stochastic bilinear equations to the problem of laser beam propagation in a turbulent medium, and the associated random Schroedinger equation. The white noise theory is shown to provide a consistent self-contained model for the Markov approximation of the refractive index field, and in particular avoids invoking ad hoc Stratanovich correction terms.

1. Introduction. The problem of evaluating the phase and amplitude fluctuations of a laser beam in refractive-index turbulence is important to many applications, in particular to adaptive optics correction. There is an extensive literature on the subject; a recent review is presented in [8]. The treatise of Tatarskii [6] is a recognized landmark reference on the subject.

In this paper we study the so-called Markov approximation [6] to the turbulence-field which we model by the finitely additive white noise theory initiated by the author [1]. Previous mathematical treatments–notably [7] –use an Ito model with an ad hoc "Stratanovich correction" term. Although our results for the moments are not new, our technique justifies the interpretation of the results as the limiting case when the bandwidth is allowed to expand in arbitrary fashion. The stationarity of the turbulence field appears to be essential for this, as well as a degree of smoothness of the covariance.

Central to the theory is the random Schroedinger equation – a bilinear stochastic partial differential equation of the kind studied initially in [1]. The current paper can in fact be viewed as an application/extension of that work. Referring to [8] for details, we shall now review briefly the genesis of this equation.

With E denoting the electric field, the usual wave equation in the standard notation takes the form

$$\mu\epsilon\frac{\nabla^2 E}{\partial t^2} = \nabla^2 E + \nabla(E \times \nabla \log \epsilon) \tag{1.1}$$

where

$$\begin{aligned} E &= E(t, x, y, z) \\ \epsilon &= \epsilon(x, y, z) \quad \text{dielectric constant of medium} \\ \mu &= \text{permittivity, which we take to be a constant.} \end{aligned}$$

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