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A MULTIPLICITY RESULT FOR PERIODIC SOLUTIONS OF HIGHER ORDER ORDINARY DIFFERENTIAL EQUATIONS

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Abstract. Ambrosetti-Prodi type results are proved for the periodic solutions of some higher-order nonlinear differential equations. The proofs are based upon a priori estimates, degree theory and Lyapunov-Schmidt method.

Introduction. In a recent paper, Fabry, Mawhin and Nkashama [2] have considered periodic problems of the form

$$\begin{aligned} &u'' + f(x,u) = s \\ &u(0) - u(2\pi) = u'(0) - u'(2\pi) = 0 \end{aligned}$$

and have proved that if

$$f(x,u) \to +\infty$$

as $|u| \to \infty$ uniformly in $x \in [0, 2\pi]$, an Ambrosetti-Prodi type result [1] holds, namely there exists s_1 such that the above problem has no solution if $s < s_1$, at least one solution if $s = s_1$ and at least two solutions if $s > s_1$. A similar result holds for

$$\begin{aligned} &u' + f(x,u) = s \\ &u(0) = u(2\pi) \end{aligned}$$

(see [5]) and the corresponding proofs rely on a combination of techniques of lower and upper solutions and degree theory.

If we consider higher order problems

$$\begin{aligned} &u^{(m)} + f(x,u) = s \\ &u(0) - u(2\pi) = \cdots = u^{(m-1)}(0) - u^{(m-1)}(2\pi) = 0, \end{aligned} \tag{1_s}$$

($m \geq 3$), the situation becomes different as the method of upper and lower solutions is only available when $m = 1$ or 2. The aim of this note is to explore the following special case of (1_s)

$$\begin{aligned} &u^{(m)} + g(u) = s + e(x,u) \\ &u(0) - u(2\pi) = \cdots = u^{(m-1)}(0) - u^{(m-1)}(2\pi) = 0 \end{aligned} \tag{2_s}$$

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