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A GREEN'S FUNCTION FOR A TWO POINT BOUNDARY VALUE PROBLEM IN GROUNDWATER THEORY

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Abstract. A classical model of a pumped, leaky aquifer-aquitard system is reconsidered with no ad-hoc provisions placed on the basic equations of Darcian motion. The model for the flow through multi-layered porous media is known to lead to a somewhat difficult system of boundary-coupled inhomogeneous o.d.e.s. The usual ad-hoc physical assumptions amount to changing this system into a solvable, although different, *homogeneous* system. The results are to be used in situations which, of course, correspond to the added assumptions.

The results in this paper relate to the original, unmodified system of many alternating layers of aquifers and aquitards with leaky interfaces. It is shown that there is a computable, physically relevant Green's Function for the full, inhomogeneous problem. This Green's Function applies to each layer in that the only changes which are required are those which physically specify a general convective diffusion kernel for a given layer. It is believed that this construction is the first of this type available for this classical problem in hydrology. Recent works have previously referred to the problem considered here as "intractable".

I. Purpose. The purpose of this article is to describe a certain layered, boundary-coupled system of ordinary differential equations and, specifically, the construction of a Green's Kernel which provides for their solution. The system and associated boundary conditions arise in the study of the groundwater hydrodynamics of pumped flow through porous media having a cylindrical geometry with permeable horizontal layers and boundaries. This basic problem, which is frequently referred to as the "leaky aquifer-aquitard problem" with a partially penetrating pump in the Hydrology literature, is a consequence of the formulation of a model for the hydraulic pressure using typical principles of mixed flow together with the empirical Darcy's Law [8]. Since the derivation of this problem has been available for many years from principal sources, there will be no duplication here; instead, reference is made to the fundamental hydrological modeling work of Hantush-Jacob [9, 10], Neuman-Witherspoon [13, 14], the texts [4, 8], and a very recent paper of Chen, et al. [6].

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