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# A DIFFUSIVE EPIDEMIC MODEL ON A BOUNDED DOMAIN OF ARBITRARY DIMENSION

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**Abstract.** We apply recent work of J. Morgan to obtain a global existence for a system of reaction diffusion equations modelling the spread of an infectious disease within a population. We then investigate the longtime behavior of the system. These results extend recent results of Webb to domains of arbitrary spatial dimension and treat the case of varying diffusion in each component.

**1. Introduction.** In this note, we shall be concerned with a system of reaction diffusion equations which describe the spread within a population of infectious disease. The population is assumed to be divided into three classes: The susceptible class S, which consists of individuals capable of becoming infected; the infective class I, which consists of individuals capable of transmitting the disease; and the removed case R, which consists of individuals who have died or have recovered from the disease and have become immune.

We make the following assumptions concerning the model: individuals of the susceptible class enter the infective class at a rate proportional to the product of the size of the susceptible class and the infective class with constant of proportionality $a \geq 0$. We assume that individuals who do not survive as infectives enter the removed class at a rate proportional to the size of the class I with a constant of proportionality $\lambda \geq 0$. Finally, we assume that diffusion takes place in all classes and that for each, the diffusion process is approximated by a constant times the Laplacian. We assume that the spatial region in which this epidemic occurs is a bounded Lipschitz domain in $\mathbf{R}^n$ with $C^{2+\alpha}$ boundary, $0 < \alpha < 1$, and that the population is constrained to remain for all time. These assumptions lead to the following weakly coupled system of semilinear partial differential equations for the population density functions:

$$\partial_t S(x,t) = d_1 \Delta S(x,t) - aS(x,t)\, I(x,t) \qquad t > 0;\ x \in \Omega \tag{1.1a}$$

$$\partial_t I(x,t) = d_2 \Delta I(x,t) + aS(x,t)\, I(x,t) - \lambda I(x,t) \qquad t > 0;\ x \in \Omega \tag{1.1b}$$

$$\partial_t R(x,t) = d_3 \Delta R(x,t) + \lambda I(x,t) \qquad t > 0;\ x \in \Omega \tag{1.1c}$$

$$\partial S(x,t)/\partial n = \partial I(x,t)/\partial n = \partial R(x,t)/\partial n = 0 \qquad t > 0;\ x \in \Omega \tag{1.1d}$$

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