Differential and Integral Equations, Volume 2, Number 2, April 1989, pp. 170-182.

POSITIVITY AND A PRINCIPLE OF LINEARIZED STABILITY FOR DELAY-DIFFERENTIAL EQUATIONS

MARY E. PARROTT
Department of Mathematics, University of South Florida, Tampa, Florida 33620 USA

(Submitted by : G.F. Webb)

Abstract. A principle of linearized stability is developed for the abstract delay differential equation $\dot{u}(t) = Bu(t) + \phi u_t$, $u_0 = f$, where B generates a strongly continuous linear semigroup and ϕ is Lipschitz continuous, and Fréchet- differentiable at an equilibrium point. Recent stability results for positive linear semigroups can then be used to obtain stability information for this equation.

I. Introduction. Let X be a Banach space with norm $\|\cdot\|$. For a fixed positive constant r_0, let $E = C([-r_0, 0], X)$ be the space of continuous functions mapping the interval $[-r_0, 0]$ to X. For $f \in E$, let $\|f\|_E = \sup_{s\in[-r_0,0]} \|f(s)\|$. We consider the abstract delay-differential equation

$$\begin{aligned} &\dot{u}(t) = Bu(t) + \phi u_t, \qquad t \geq 0 \\ &u_0 = f. \end{aligned} \tag{FDE}$$

Here we assume that B generates a strongly continuous semigroup of bounded linear operators on X, and ϕ is a nonlinear Lipschitz continuous operator from E to X. (These hypotheses are made more precise later.) The functional u_t is defined by translation as $u_t(s) = u(t+s)$ for $s \in [-r_0, 0]$.

Equations which can be written in the abstract form (FDE) often serve as models for various phenomena where a certain state at time t depends on information about the state at an earlier time. Such equations appear in many diverse areas of physical and biological sciences.

Our goal is to use recent results from the theory of positive semigroups in order to study the stability properties of (FDE). Specifically, we will use a result of W. Desch and W. Schappacher [2] to develop a "principle of linearized stability" for (FDE) (under the additional assumption that ϕ is Fréchet- differentiable at an equilibrium point), and then use known stability results for positive linear semigroups in order to obtain stability information for (FDE).

Received June 1, 1988.
Supported in part by National Science Foundation Grant NSF#DMS-8706798.
The author wishes to express her thanks to Professor Glenn Webb for many helpful discussions, and to the Department of Mathematics for an enjoyable year at Vanderbilt University.
AMS Subject Classifications: 34K20, 34K30.