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DIRICHLET-TYPE CRITERIA AND ACCRETIVENESS CRITERIA FOR COMPLEX STURM-LIOUVILLE OPERATORS

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(Submitted by: W.N. Everitt)

Dedicated to Professor W.N. Everitt on the occasion of his 65th birthday

Abstract. Second order differential expressions of the form $w^{-1}(-(pf')' + f)$ are considered on a real interval I with one singular end point, assuming $w > 0$ is real-valued but permitting both p and q to be complex-valued. A number of criteria are established for the expression to be Dirichlet, conditional Dirichlet and strong limit-point. These lead to criteria for certain associated operators in the weighted Hilbert space $L^2_w(I)$ to be maximal dissipative or maximal quasi-accretive.

I. Introduction. This paper is concerned with the second order, ordinary quasi differential expression τ defined for suitable, complex-valued functions f, by

$$\tau f = w^{-1}(-(pf')' + f) \quad \text{on } [a,b), \tag{1.1}$$

where p, q, and w satisfy (1.2), (1.3) below, and $[a,b)$ is an interval of the real line, with $-\infty < a < b \leq +\infty$. The terms limit-point (LP), strong limit-point (SLP), Dirichlet (D) and conditional Dirichlet (CD) at b are often used to describe certain properties associated with τ. In Race [20], the connections between them were discussed under very general assumptions concerning w and complex-valued p and q. For the history of the use of these terms, especially for complex-valued coefficients, see [20, section 2] and the references cited there. For an early review of D, CD and SLP criteria for real-valued coefficients (when $w = 1$), see Everitt [6].

It is well known that for real-valued p and q, the above properties often relate to operators associated with τ being bounded below. Correspondingly, for complex-valued p and q, they relate to such operators being accretive or dissipative (see, for example [15]). There is also a close connection with certain integral inequalities (see [5] and the references contained therein).

The purpose of the present work is to provide conditions on the coefficients in (1.1) which will ensure τ is Dirichlet or conditional Dirichlet. In each case this will be shown to ensure that τ is strong limit-point. It is also shown that in most cases this leads to certain associated operators being maximal accretive or dissipative.

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