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Erratum

Volume 2, Number 1 (1989), in the article "Multiple Periodic Solutions of some Nonlinear Ordinary Differential Equations of Higher Order," by Miguel Ramos and Luis Sanchez, pages 81-90:

The argument in Theorem 3 which provides the contradiction

$$J(v+\epsilon(\phi-b)) < J(v)$$

does not apply. Indeed, the function ϕ depends on s and therefore, we cannot deduce that the coefficient of ϵ is negative.

The problem of the validity of Theorem 3 remains open. We replace it by a somewhat related theorem.

3. A case where g has polynomial growth. In this section we use a minimization argument to partially extend Theorem 1. Precisely, we take m to be an even integer and consider the variational problem

$$(-1)^{\frac{m}{2}} u^{(m)} + g(t,u) = s, \quad u^{(i)}(0) = u^{(i)}(2\pi), \;\; i = 0,1,\cdots,m-1. \qquad (P')$$

Assume the following hypothesis on g.

(H3). There exist positive numbers A, A', C, C' R and $p \geq 2$, such that, if we let $q = p/(p-1)$ and $G(t,u) = \int_0^u g(t,x)\,dx$, we have

$$|G(t,u)| \leq \begin{cases} A\frac{|u|^p}{p} + A' & \text{if } (t,u) \in [0,2\pi]\times[R,+\infty), \\ C\frac{|u|^q}{q} + C' & \text{if } (t,u) \in [0,2\pi]\times(-\infty,-R]. \end{cases}$$

Theorem 3. *Let $g(t,u)$ be a continuous and positive function defined on $[0,2\pi]\times\mathbb{R}$, satisfying (H) and $(H3)$. Assume further that*

$$C < A^{1-q}(2\pi)^{q/2} + \frac{q}{2}(2\pi)^{(q-2)/2}.$$

Then problem (P') has property B.

Proof: Consider, in the Hilbert space $X = H_{2\pi}^{\frac{m}{2}}$ (with norm given by $(|u|_2^2 + |u^{(\frac{m}{2})}|_2^2)^{\frac{1}{2}}$), the functional

$$I(u) = \int_0^{2\pi} [\frac{u^{(j)^2}}{2} + G(t,u) - su]\,dt.$$

Here we have written $j = m/2$. This functional is, of course, of class C^1 and its critical points in X are precisely the solutions of (P'); moreover, I is weakly lower semicontinuous. For each $s > 0$, introduce the closed and convex subset of X

$$K = \{v \in X : \bar{v} \geq 0 \text{ and } |v^{(j)}|_2 \leq 2\pi s\},$$