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NON-EXISTENCE OF RADIALLY SYMMETRIC NON-NEGATIVE SOLUTIONS FOR A CLASS OF SEMI-POSITONE PROBLEMS

K.J. Brown
Department of Mathematics, Heriot Watt University, Edinburgh, EH14 4AS, Scotland

Alfonso Castro†
Department of Mathematics, University of North Texas, Denton, Texas 76203 USA

R. Shivaji‡
Department of Mathematics, Mississippi State University, Mississippi State, MS 39762 USA

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Abstract. We study the radially symmetric non-negative solutions for the boundary value problem

$$-\Delta u(x) = \lambda f(u(x)); \quad \| x \| < 1, \ \ x \in R^N \quad (N \geq 2)$$
$$u(x) = 0; \quad \| x \| = 1$$

where $\lambda > 0$, $f(0) < 0$ (semi-positone) and f superlinear. We establish that there exists a $\lambda_0 > 0$ such that for $\lambda > \lambda_0$ there are no non-negative solutions.

1. Introduction. Here we consider the radially symmetric non-negative solutions for the boundary value problem

$$-\Delta u(x) = \lambda f(u(x)); \quad \| x \| < 1, \quad x \in R^N, \ \ N \geq 2 \tag{1.1}$$
$$u(x) = 0; \quad \| x \| = 1 \tag{1.2}$$

where $\lambda > 0$ and $f : [0, \infty) \to R$ is such that $f' \geq 0$. It is well known that the study of (1.1)-(1.2) is equivalent to

$$-u'' - (n/r)u' = \lambda f(u); \quad 0 < r < 1 \tag{1.3}$$
$$u'(0) = 0 \tag{1.4}$$
$$u(1) = 0 \tag{1.5}$$

where $n = N - 1$. We will assume that there exists $\alpha > 1$ such that

$$\liminf_{u \to \infty} f(u)/u^\alpha > 0 \tag{1.6}$$

and

$$f(0) < 0. \tag{1.7}$$

Our main result is given in Theorem 1.1.

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