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THE KELDYS-FICHERA BOUNDARY VALUE PROBLEMS FOR DEGENERATE QUASILINEAR ELLIPTIC EQUATIONS OF SECOND ORDER*

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In this paper, we establish an acute angle principle for weakly continuous mappings, and then as an application, we discuss the Keldys-Fichera boundary value problem of degenerate quasilinear elliptic equations of second order. We also discuss the maximum principle, the comparison principle and the modular estimate theorem for nonlinear equations with nonnegative characteristic form.

In [1] O.A. Oleinik and E.V. Radkevich have made a detailed summary for the Keldys-Fichera boundary value problem for linear equations with nonnegative characteristic form of second order. But little information is known for Keldys-Fichera boundary conditions for nonlinear equations with nonnegative characteristic form.

1. The acute angle principle for weakly continuous operators. Let X be a linear space, and X_1, X_2 be completions of X under the norms $\|\cdot\|_1$ and $\|\cdot\|_2$, respectively. Assume that X_1 is a reflexive Banach space and X_2 is a separable Banach space. The symbols "$\rightharpoonup$" and "$\rightarrow$" denote weak convergence and strong convergence, respectively.

Definition 1.1. *Let Y_1, Y_2 be two Banach spaces. A mapping $G : Y_1 \rightarrow Y_2^*$ is called weakly continuous if, for any x_n, $x_0 \in Y_1$, $x_n \rightharpoonup x_0$, there exists a subsequence $\{x_{n_k}\}$ such that*

$$\lim_{k\to\infty} \langle Gx_{n_k}, y\rangle = \langle Gx_0, y\rangle, \quad \text{for } y \in X_2.$$

We introduce the *acute angle principle of* Brouwer *degree.*

Lemma 1.2. *Let $F : \bar{\Omega} \rightarrow R^m$ be a continuous mapping, $\Omega \subset R^m$ be a bounded open set, $0 \in \Omega$. If*

$$\langle Fx, x\rangle \geq 0, \quad \text{for } x \in \partial\Omega,$$

then the equation $Fx = 0$ has a solution in $\bar{\Omega}$.

Proof: Suppose $Fx \neq 0$ for any $x \in \partial\Omega$, otherwise the theorem is true. For any $t \in [0,1]$, $x \in \partial\Omega$, we have

$$tx + (1-t)Fx \neq 0. \tag{1.2}$$

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