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ON THE SOLVABILITY OF ONE CLASS OF BOUNDARY VALUE PROBLEMS

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Consider the vector differential equation

$$\ddot{r} = f(t, r, \dot{r}) \tag{1}$$

and the corresponding boundary value problem

$$r(0) = 0, \qquad r(\rho) = r_c \tag{2}$$

$$\| \dot{r}(0) \| = v_0, \tag{3}$$

where $\rho > 0$ is not fixed, and the vector r_c and scalar $v_0 > 0$ are given. The problem of solvability of boundary value problem (2), (3) for the equation (1) is under consideration in this paper.

This problem has the wide scope of the corresponding technical and physical applications because it may be interpreted in the following way: there is the source of particles with fixed energy (the boundary condition (3)) at the point $r = 0$. These particles are moving according to the known physical field $f(t, r, \dot{r})$. The question is: under what conditions will the particle from this source attend the given point $r = r_c$? Here the time of moving ρ is not fixed.

Nevertheless in the case when the equation (1) is not integrated, the theory of solvability of this problem has not yet been given. The main idea of this paper is to formulate the sufficient conditions for solvability of boundary value problem (2), (3) for the equation (1) and to present the procedure of constructing the corresponding solution. This paper is in final form and no version of it will be submitted for publication elsewhere.

According to the change of the variables $x = r - (r_c \cdot t)/\rho$ and $t = \rho \cdot \tau$, the initial boundary value problem reduces to the equivalent problem:

$$\ddot{x} = \rho^2 f\big(\rho \cdot \tau,\ x + r_c \cdot \tau,\ \frac{\dot{x} + r_c}{\rho}\big) = \phi(\rho, x, \dot{x}, \tau) \tag{4}$$

$$x(0) = 0, \quad x(1) = 0 \tag{5}$$

$$\| \dot{x}(0) + r_c \| = v_0 \cdot \rho. \tag{6}$$

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