Differential and Integral Equations, Volume 3, Number 1, January 1990, pp. 161–173.

SCATTERING THEORY FOR HIGHER ORDER EQUATIONS

DOUGLAS PICKETT

Department of Mathematics, Tulane University, New Orleans, LA 70118 USA

(Submitted by: J.A. Goldstein)

Abstract. This paper deals with scattering theory for equations of the form $\prod_{k=1}^{M}(\frac{d}{dt} - B_k)u(t) = 0$, where $M = 2^N$ with N a positive integer, and $\{B_k\}$ are M commuting, skew-adjoint operators on a Hilbert space $\mathcal{H}$. It has been shown [3] that equations of this form may be written $\frac{d}{dt}v(t) = \mathbf{A}v(t)$ on $\mathcal{H}^M$, where $\mathbf{A}$ is a skew-adjoint, M by M matrix of operators on the Hilbert space $\mathcal{H}^M$. This matrix will be shown to be unitarily equivalent to the $M \times M$ diagonal matrix with the operators $\{B_k\}$ on the diagonal. If $\{B_k^n\}$, $n = 0, 1$, are two families of commuting, skew-adjoint operators on $\mathcal{H}$, let $\mathbf{A}^n$, $n = 0, 1$, be the matrices of operators on $\mathcal{H}^M$ corresponding to $\{B_k^n\}$, $n = 0, 1$. It will be shown that the wave operators $\Omega_\pm(iB_k^1, iB_k^0)$ on $\mathcal{H}$ exist (and are complete) for $k = 1, 2, \dots, M$, if and only if the wave operators $\Omega_\pm(i\mathbf{A}^1, i\mathbf{A}^0)$ exist (and are complete). This result will be applied to the equations of linear elasticity.

1. Introduction. Given a pair of self-adjoint operators H_1, H_0 on a Hilbert space $\mathcal{H}$, let $\mathcal{H}_{j,ac}$, $j = 0, 1$, denote the subspace of absolute continuity for H_j, $j = 0, 1$, and let P_j^{ac} be the orthogonal projection onto it. The *wave operators* $W_\pm(H_1, H_0)$ are defined by

$$W_\pm(H_1, H_0)f = \lim_{t \to \pm\infty} e^{itH_1} e^{-itH_0} P_0^{ac} f, \quad \text{for } f \in \mathcal{H},$$

provided this limit exists. (See [4] for details.)

If the wave operators exist and are complete (i.e., Range $W_\pm(H_1, H_0) = \mathcal{H}_{1,ac}$), the *scattering operator*

$$S(H_1, H_0) = W_+^*(H_1, H_0) W_-(H_1, H_0)$$

is a unitary operator on $H_{0,ac}$.

This describes the case in which the "simple, unperturbed" equation,

$$i\frac{du_0}{dt} = H_0 u_0(t) \qquad (t \in \mathbf{R}),$$

Received November 12, 1988.
AMS Subject Classifications: 35L30, 35P25, 47A40, 73C99.