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EXISTENCE OF FORCED OSCILLATIONS FOR SOME SINGULAR DYNAMICAL SYSTEMS

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Abstract. In this work, we search for T-periodic solutions of the second-order Hamiltonian system $u'' + V'(u) = f(t)$, where $f(t)$ is a T-periodic forcing term, and the potential V goes to $-\infty$ at a point or on a linear subspace of $\mathbb{R}^N$. Under a suitable "strong force" assumption on V, the methods of critical point theory can be utilized to get various existence results.

Section 0. In this paper, we study the existence of periodic solutions, with a fixed period T, of the system

$$u'' + V'(u) = f(t), \tag{0.1}$$

where $V \in C^1(\Omega, \mathbb{R})$, Ω is an open subset of $\mathbb{R}^N$, and $f \in C(\mathbb{R}, \mathbb{R}^N)$ is T-periodic. We assume that

$$\lim_{x \to \partial\Omega} V(x) = -\infty, \tag{0.2}$$

and we will call $\partial\Omega$ "the set of singularities" of the system (0.1). This kind of problem was considered by variational methods for the first time by Gordon [6], and, more recently, by many authors (see [1–5], [7] and their bibliographies).

In this work, we set $\mathbb{R}^N = \mathbb{R}^n \oplus \mathbb{R}^m$, with $n \geq 3$ and $m \geq 0$, and we suppose that $\Omega = \mathbb{R}^N \backslash \mathbb{R}^n$, so the set of singularities of (0.1) is, in our case, $S \equiv \partial\Omega = \mathbb{R}^m$ (the case $S = \{0\}$ is not excluded); in this situation the effect of the periodic forcing term $f(t)$ is studied. We assume that the "strong force" assumption holds (see [6]):

$$\text{there exists } U \in C^1(\Omega, \mathbb{R}) \text{ such that:} \quad \text{i) } \lim_{x \to \partial\Omega} U(x) = -\infty \quad \text{ii) } -V(x) \geq |U'(x)|^2 \text{ in a suitable neighborhood of } S. \tag{0.3}$$

Moreover we assume

$$V < 0, \quad \text{and} \quad V(x),\ V'(x) \to 0 \text{ as } \operatorname{dist}(x, S) \to \infty. \tag{0.4}$$

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