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DYNAMIC THEORY OF QUASILINEAR PARABOLIC EQUATIONS II. REACTION-DIFFUSION SYSTEMS

HERBERT AMANN

Mathematisches Institut, Universität Zürich, Rämistrasse 74, CH–8001 Zürich, Switzerland

(Submitted by: M.G. Crandall)

Introduction. Let Ω be a bounded smooth domain in $\mathbb{R}^n$ and consider a second order differential equation of the form

$$\partial_t u - \partial_j(a_{jk}(\cdot, u)\partial_k u) = f(\cdot, u, \partial u) \quad \text{on} \quad \Omega \times (0, \infty) \tag{1}$$

acting on $\mathbb{R}^N$-valued functions $u = (u^1, \dots, u^N)$. (We use the summation convention throughout, j and k running from 1 to n, and r and s running from 1 to N.) We assume that

$$a_{jk} \in C^\infty(\overline{\Omega} \times G, \mathcal{L}(\mathbb{R}^N)), \quad 1 \le j, k \le n,$$

where G is an open subset of $\mathbb{R}^N$ and $\mathcal{L}(\mathbb{R}^N)$ is the space of all real $N \times N$ matrices. We assume also that

$$f \in C^\infty(\overline{\Omega} \times G \times \mathbb{R}^{nN}, \mathbb{R}^N)$$

and that f is 'affine in the gradient', that is,

$$f(\cdot, \cdot, \eta) = f_0 + f_j \eta_j, \quad \eta := (\eta_1, \dots, \eta_n) \in \mathbb{R}^N \times \dots \times \mathbb{R}^N,$$

where $f_0 : \overline{\Omega} \times G \to \mathbb{R}^N$ and $f_j : \overline{\Omega} \times G \to \mathcal{L}(\mathbb{R}^N), 1 \le j \le n$.

Equation (1) has to be complemented by boundary conditions, which are typically 'Dirichlet boundary conditions',

$$u = 0 \quad \text{on} \quad \partial\Omega \times (0, \infty), \tag{2}$$

or 'Neumann type boundary conditions',

$$a_{jk}(\cdot, u)\nu^j \partial_k u = g(\cdot, u) \quad \text{on} \quad \partial\Omega \times (0, \infty), \tag{3}$$

where $\nu := (\nu^1, \dots, \nu^n)$ is the outer unit normal vector field on $\partial\Omega$ and

$$g \in C^\infty(\partial\Omega \times G, \mathbb{R}^N).$$

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