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COMPARISON OF EIGENVALUES FOR FOCAL POINT PROBLEMS FOR n-th ORDER DIFFERENCE EQUATIONS

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Abstract. Existence and comparison theorems for eigenvalues of $(k, n-k)$-focal boundary value problems are given for a class of nth order difference equations. The techniques involve the theory of operators on a Banach space. Typically, "positivity" conditions are placed on the coefficient functions in the difference equation in order that related operators be u_0-positive. In addition, we show how these requirements can be relaxed and still obtain existence and comparison results.

1. Introduction. We are mainly interested in proving the existence of a smallest positive eigenvalue for the focal boundary value problem (1), (3) below and in proving comparison theorems for the focal boundary value problems

$$\Delta^{n-k}[a(t)\Delta^k u(t)] = (-1)^{n-k}\lambda\Big\{\sum_{i=0}^{k-1} p_i(t)\Delta^i u(t) + \sum_{i=0}^{n-k-1} p_{k+i}(t)\Delta^i[a(t)\Delta^k u(t)]\Big\} \tag{1}$$

and

$$\Delta^{n-k}[A(t)\Delta^k u(t)] = (-1)^{n-k}\Lambda\Big\{\sum_{i=0}^{k-1} P_i(t)\Delta^i u(t) + \sum_{i=0}^{n-k-1} P_{k+i}(t)\Delta^i[A(t)\Delta^k u(t)]\Big\} \tag{2}$$

subject to the boundary conditions

$$\begin{aligned} \Delta^i u(\alpha) &= 0, \qquad 0 \le i \le k-1, \\ \Delta^{k+i} u(\beta+1) &= 0, \quad 0 \le i \le n-k-1, \end{aligned} \tag{3}$$

under suitable conditions on the coefficients. Here, Δ is the difference operator defined by $\Delta y(t) = y(t+1) - y(t)$ and t is a discrete variable taking on the integer values $\alpha \le t \le \beta + n$. We assume throughout that $\beta \ge \alpha + k$ and $a(t) > 0$, $A(t) > 0$

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