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FREE BOUNDARY PROBLEMS FOR EMDEN–FOWLER EQUATIONS*

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Abstract. This article is concerned with free boundary problems for differential equations of the Emden-Fowler type,

$$u'' + g(x)u' + f(u) = 0, \quad x > 0.$$

In particular, it addresses the questions of existence and uniqueness of a finite point P and a solution u satisfying the conditions

$$u'(0) = 0;\ u(x) > 0,\ 0 < x < P;\ u(P) = u'(P) = 0.$$

The classes of admissible functions f and g include functions of the type $f(u) = u^p - u^q$ and $g(x) = m/x$, where $0 \le q < p \le 1$ and $m > 0$. A special instance, where $f(u) = \sqrt{u} - 1$ and $g(x) = 1/x$, has been proposed in plasma physics as a simple model for Tokamak equilibria with magnetic islands.

1. Introduction. In this article we study free boundary problems for the Emden-Fowler differential equation

$$u'' + g(x)u' + f(u) = 0, \quad x > 0. \tag{1}$$

We prove that, for broad classes of functions f and g, there exists a finite point P and a solution u of (1) satisfying the conditions

$$u'(0) = 0;\ \ u(x) > 0,\ \ 0 < x < P;\ \ u(P) = u'(P) = 0. \tag{2}$$

The classes of admissible functions include functions of the type $f(u) = u^p - u^q$ and $g(x) = m/x$, where $0 \le q < p \le 1$ and $m > 0$. A special instance, where $f(u) = \sqrt{u} - 1$ and $g(x) = 1/x$, has been proposed recently by Miller, Faber, and White [1] in plasma physics as a very simple model of Tokamak equilibria with magnetic islands, and provided the motivation for the present investigation.

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