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A DIFFUSION SYSTEM FOR FLUID IN FRACTURED MEDIA†

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Abstract. When modeling diffusion phenomena, equations of the form $\frac{d}{dt}A(u)+B(u)=f$ often arise. When both A and B are non-linear, many of these diffusion problems remain unsolved. In this paper a class of such problems is presented and solutions are found. Properties of these solutions are also considered.

1. Introduction. The class of problems we consider may be motivated by the physical problem associated with the flow of a fluid (liquid or gas) through fractured porous rock. In such a situation, the majority of the bulk fluid motion will be in the fissures between the blocks, and the block to block diffusion is small. However, the blocks will store and release fluid as the pressure in the fissures changes. Since the block volume will be much larger than the fissures volume, the block to fissure diffusion is a major component of the system.

If the usual continuum assumptions are made, the balance of mass in the blocks and fissures may be written as

$$\begin{aligned} &\frac{\partial}{\partial t}(m_B\rho_B) = m_B f - a\gamma \\ &\frac{\partial}{\partial t}(m_F\rho_F) + \nabla\cdot(m_F\rho_F\mathbf{U}_F) = m_F g + a\gamma \end{aligned} \tag{1.1}$$

where m_B, m_F are the ratios of block and fissure volumes to the total volume, γ is the mass flow rate of fluid exchanged between the blocks and fissures, a is a geometric quantity describing the ratio of the block-fissure surface area per unit volume, ρ_B, ρ_F are the densities of the fluid in the block and fissure, f, g are the fluid sources in the blocks and fissures respectively, $\mathbf{U}_F$ is the bulk fluid velocity in the fissures.

The porous flow assumption (d'Arcy's law) determines the velocity as

$$\mathbf{U} = -\sum_{j=0}^{n} \kappa_j\left(\rho\frac{\partial p}{\partial x_j}\right)\frac{\partial p}{\partial x_j}\,\mathbf{e}_j$$

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