Differential and Integral Equations, Volume 3, Number 3, May 1990, pp. 541–570.

PERIODIC SOLITON SOLUTIONS TO NONLINEAR KLEIN-GORDON EQUATIONS ON $\mathbb{R}^2$

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(Submitted by: L. Salvadori)

Abstract. In this paper, we prove the existence and the regularity of time-periodic free vibrations to certain nonlinear Klein-Gordon equations in one space dimension. Those vibrations decay toward a constant equilibrium solution as the spatial variable goes to plus or minus infinity, and their spectrum consists of integral multiples of some prescribed frequency. Using certain uniqueness and symmetry arguments, we also prove the nonexistence of small amplitude homoclinic orbits for a certain dynamical system associated with the given Klein-Gordon equations. We finally apply our existence and nonexistence results to several semilinear wave equations of current interest in theoretical physics. In particular, we provide a complete proof of a local version of the EKNS-conjecture for nonlinear Klein-Gordon equations with polynomial nonlinearities. We also prove the nonexistence of small amplitude breathers in the Φ^4-theory, a fact that bears some analogy with, but does not seem to be completely equivalent to, some recent results of Kruskal and Segur [12].

1. Introduction and outline. The central theme of this article is concerned with a mathematical analysis of certain real solutions to nonlinear Klein-Gordon equations of the form

$$u_{tt}(x,t) = u_{xx}(x,t) - g(u(x,t)) \tag{1.1}$$

where $(x,t) \in \mathbb{R}^2$ and $g : \mathbb{R} \to \mathbb{R}$. Equations of the type (1.1) occur in various contexts, such as in the description of wave propagation phenomena in superconductors, ferromagnets and nonlinear optics (with $g(u) = \sin(u)$ or $g(u) = \sin(u) + \sin(u/2)$), in the theory of dislocation of crystals (for example with $g(u) = u - u^2 - u^3$) and in the classical modelling of certain phenomena in field theory (with $g(u) = u^3 - u$) ([1]–[6]). The relevance of equations of the form (1.1) to those fields mainly lies in the belief that they can, for some g's, exhibit the so-called soliton bound state

Received January 16, 1989.
Work partly supported by the Swiss National Science Foundation under Grant 2.002/0.86 and by the "Gruppo Nazionale per la Fisica Matematica" of the Italian Research Council (CNR).
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AMS Subject Classifications: 35L05, 35L15, 46N05.