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ALMOST PERIODIC SOLUTIONS OF A LINEAR VOLTERRA SYSTEM*

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(Submitted by: C. Corduneanu)

Dedicated to Professor Tosihusa Kimura on his 60th birthday

Abstract. The purpose of this paper is to discuss the existence of almost periodic solutions of a system of linear Volterra equations by assuming uniform asymptotic stability in the sense of uniform norm of the zero solution of its homogeneous system. It is shown that the above stability property induces a special case of Amerio's separation condition.

We shall show the existence of almost periodic solutions of a system of linear Volterra equations

$$\dot{x}(t) = D(t)x(t) + \int_{-\infty}^{t} E(t,s)x(s)\,ds + f(t), \quad t \in \mathbb{R}, \tag{1}$$

equivalently,

$$\dot{x}(t) = D(t)x(t) + \int_{-\infty}^{0} E(t,t+s)x(t+s)\,ds + f(t), \tag{1'}$$

where D is an $n \times n$ matrix of functions continuous on $\mathbb{R}$, $\mathbb{R} = (-\infty, \infty)$, E is an $n \times n$ matrix of functions continuous for $-\infty < s \leq t < \infty$ and f is an n-vector of functions continuous on $\mathbb{R}$. Denote by $|x|$ the Euclidean norm of $x \in \mathbb{R}^n$ and set $|A| = \sup\{|Ax| : |x| \leq 1\}$ for any $n \times n$ matrix A.

We assume the following conditions throughout this paper:

(A) $D(t)$, $f(t)$ and $E(t,t+s)$ are almost periodic, where $E(t,t+s)$ is said to be almost periodic in t uniformly for s, if for any $\epsilon > 0$ and any compact set $K \subset I^-$, $I^- = (-\infty, 0]$, there exists a positive number $L(\epsilon, K)$ such that any interval of length $L(\epsilon, K)$ contains a τ for which $|E(t,t+s) - E(t+\tau, t+\tau+s)| < \epsilon$ for all $t \in \mathbb{R}$ and $s \in K$. For the properties of an almost periodic function with parameters, see [9].

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