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EXISTENCE THEOREMS FOR ELLIPTIC MONGE-AMPÈRE EQUATIONS IN THE PLANE

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Abstract. Two-dimensional Monge-Ampère equations of the form

$$D_{11}uD_{22}u - (D_{12}u)^2 - \alpha(D_{11}u + D_{22}u) = f(|x|, u, |\nabla u|), \tag{1}$$

$\alpha \geq 0$ being a constant, are studied in the entire plane $\mathbb{R}^2$. By application of the Schauder-Tychonov fixed point theorem to an integro-differential operator associated with (1) sufficient conditions are given under which (1) has infinitely many positive radially symmetric entire solutions which are strictly convex in $\mathbb{R}^2$ and asymptotic to constant multiples of $|x|$ (if $\alpha = 0$) or $|x|^2$ (if $\alpha > 0$) as $|x| \to \infty$.

1. Introduction. Our purpose is to establish the existence of positive, radial, strictly convex solutions $u \in C^2(\mathbb{R}^2)$ of radially symmetric two-dimensional Monge-Ampère equations

$$D_{11}uD_{22}u - (D_{12}u)^2 - \alpha(D_{11}u + D_{22}u) = f(|x|, u, |\nabla u|), \quad x \in \mathbb{R}^2, \tag{1}$$

where α is a nonnegative constant, $f \in C(\overline{\mathbb{R}}_+ \times \mathbb{R}_+ \times \overline{\mathbb{R}}_+)$, $\mathbb{R}_+ = (0, \infty)$, $\overline{\mathbb{R}}_+ = [0, \infty)$. As usual, $|x|$ denotes the Euclidean length of $x = (x_1, x_2)$, $D_i = \partial/\partial x_i$, $D_{ij} = D_iD_j$ for $i, j = 1, 2$, and $\nabla = (D_1, D_2)$. Additional hypotheses on f are described in §§2 and 3.

Monge-Ampère equations have been the topic of numerous investigations in geometry and analysis for two centuries, and lately there has been renewed interest [2-8, 11]. The theoretical development originated mainly from various problems of geometry, whose analytic formulations require consideration of partial differential equations of type (1). Typical examples of such geometric problems are: (i) The problem of constructing surfaces in $\mathbb{R}^3$ with prescribed Gaussian curvature; and (ii)

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