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A THREE–POINT BOUNDARY VALUE PROBLEM

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Abstract. We report preliminary results about boundary value problem

$$x''' = f(t,x) \quad \text{on } [0,1], \;\; x(0) = x(1) = x'(\eta) = 0 \text{ with } \eta \in [0,1]$$

and concentrate on $\eta = 1/2$, the only case where $\lambda = 0$ is an eigenvalue of the linear problem.

1. Introduction. Let us first recall that monograph [7] contains, in particular, a lot of basic material on third order equations, while [4] gives some additional information about the linear case, including some simple three point problems and some motivation from engineering in Chap. IV. Recently, the literature on this kind of boundary conditions has grown; see e.g. [1], [5] and the references given there. In the main one has considered problems which can be solved by matching the results known for the two individual intervals or by choosing boundary conditions which give a priori bounds for solutions easily. For $\eta \in (0,1)$ the problem mentioned above is not of this type, and $\eta = 1/2$ is technically the worst case since already $\lambda = 0$ is an eigenvalue. As far as we know this has not been discussed in the literature. Therefore, we first consider the linear eigenvalue problem, determine Green's function if λ is not an eigenvalue, and discuss other methods which may give bounds for the solution in terms of the right hand side, up to the point where it comes to "heavy" computations which should be done by somebody else with more numerical interest and the right equipment. Finally, we briefly indicate how this linear information can be used for the nonlinear problem.

2. The abstract formulation. Let $J = [0,1]$ and $X = L^2(J)$ with $|x| = (x,x)^{1/2}$, where $(x,y) = \int_0^1 x(t)y(t)\,dt$. Then boundary value problem

$$x''' = f(t,x) \quad \text{a.e. in } J, \tag{1}$$

$$x(0) = x(1) = x'(\tfrac{1}{2}) = 0 \tag{2}$$

will be written as $Lx = Fx$ with $(Fx)(t) = f(t,x(t))$, $Lx = x'''$ and $D_L = \{x : x''' \in X \text{ and } (2)\}$. We have $N(L) = \{x \in D_L : Lx = 0\}$ =span ϕ, where

$$\phi(t) = \sqrt{30}\,t(1-t) \quad \text{and} \quad |\phi| = 1. \tag{3}$$

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