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MULTIPLICITY RESULTS FOR A SEMILINEAR ELLIPTIC PROBLEM WITH CROSSING OF MULTIPLE EIGENVALUES

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Introduction. Let $\Omega \subset \mathbb{R}^N$ be a smooth bounded domain and consider the Dirichlet problem

$$-\Delta u = f(u) + h \quad \text{in } \Omega, \quad u = 0 \text{ on } \partial\Omega \tag{0.1}$$

where $h \in L^2(\Omega)$ is a given function and $f \in C(\mathbb{R})$ satisfies

$$\lim_{s \to -\infty} f(s)/s = \beta \qquad \lim_{s \to \infty} f(s)/s = \alpha. \tag{f$_1$}$$

Let $\lambda_1 < \lambda_2 \leq \lambda_3 \leq \dots$ be the eigenvalues (not necessarily distinct) of $-\Delta$ with Dirichlet boundary condition and $\phi_1, \phi_2, \phi_3, \dots,$ the corresponding eigenfunctions. There exists an extensive literature on this problem in the case that $\alpha \neq \beta$ and some eigenvalue λ_j belongs to the interval determined by α and β (see [5] and its bibliography). In [9], Podolak showed that, if λ_j is a simple eigenvalue, $\int |\phi_j|\phi_j \neq 0$ and $\epsilon > 0$ is small enough, then $(\alpha, \beta) = (\lambda_j + \epsilon, \lambda_j - \epsilon)$ belongs to the so-called Ambrosetti-Prodi region. In this case, with additional hypotheses on f and for appropriate $h \in L^2(\Omega)$, she obtained the existence of at least two solutions of problem (0.1). The authors in [1], with slightly stronger hypotheses, showed that problem (0.1) has exactly two solutions. The authors of [6] and [10] extended the result in [9], and [11] extended the result in [1].

In this paper we consider also the case when $(\alpha, \beta) = (\lambda_j + \epsilon, \lambda_j - \epsilon)$, but allowing λ_j to have multiplicity $m > 1$. In the case that m is even and a condition on the eigenspace is satisfied, we obtain the existence of four solutions. The author is grateful to D.G. de Figueiredo for several discussions during this work, and also to the referee for the example given at the end of the paper.

1. The main result. Let λ_k and ϕ_k, $k = 1, 2, \dots,$ be as in the Introduction, and assume that $\lambda_j = \lambda_{j+1} = \dots = \lambda_{j+m-1}$ is an eigenvalue of multiplicity m. Given a direction $(a, b) \in \mathbb{R}^2$ we will study problem (0.1) for $(\alpha, \beta) = (\lambda_j + \epsilon a, \lambda_j + \epsilon b)$ and $\epsilon > 0$ sufficiently small.

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