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GLOBAL STABILITY IN DIFFUSIVE DELAY LOTKA-VOLTERRA SYSTEMS

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Abstract. We consider the global asymptotic stability of diffusive delay Lotka-Volterra systems which may model population dynamics of closed ecological environments containing n interacting species. The first part of this paper deals with discrete delay case, where both continuous and discrete diffusion situations are considered. The second part of this paper studies unbounded continuous delay cases, where the integral kernels are assumed to satisfy linear differential equations with constant coefficients. In both parts, sufficient conditions for global asymptotic stability of the unique positive steady states are derived via some proper Lyapunov functions. To some extent, these results indicate that the diffusivity of the system may not affect the global asymptotic stability of its reaction system.

1. Introduction. In this paper, we deal with the global asymptotic stability of diffusive delay Lotka-Volterra systems which may be used to model the population dynamics of a closed ecological environment containing n interacting species.

The mathematical theory of population dynamics has largely been developed by employing first-order ordinary differential equations [4, 7, 8, 10, 20]. The population growth of a species is generally assumed to respond instantaneously to the densities of its own population and the populations of other species with which it interacts. However, it is often more realistic and appropriate to allow the rates of change of the variables to depend on the previous histories of the population densities as well as the possible dispersal rates caused by the spatial heterogeneity of the environment. It is these considerations which motivate us to study the diffusive delay Lotka-Volterra system. When the environment is assumed to be homogeneous, this system reduces to the well-known delay Lotka-Volterra system which was extensively studied by many authors (see Beretta and Solimano [1], Wörz-Busekros [23], Cushing [4] and those references cited therein). In the case that population growth does not depend on previous population densities, it reduces to the well-studied reaction-diffusion systems (see Brown [3], Dunbar et al. [5], Hastings [12], Smoller [21] and those references cited therein). In this circumstance, our results reduce to the ones appearing in Dunbar, Rybakowski and Schmitt [5]. In fact, our method is a generalization of the one adopted in [5]. Since the last decade, the

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