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EXISTENCE AND UNICITY OF SOLUTIONS OF n-POINT BOUNDARY VALUE PROBLEMS

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Abstract. Assuming that $(n-1)$-point boundary value problems have unique solutions, a unique solution to a class of n-point boundary value problems is constructed. This is accomplished by a suitable Liapunov-like function and a solution matching technique. Length of interval estimates for $n = 4$, which depends on the distribution of the points within the length of interval, are also established.

1. Introduction. This paper presents a criterion for the existence and uniqueness of solutions to n-point boundary problems associated with the differential equation

$$y^{(n)} = f(x, y, y', \dots, y^{(n-1)}) \tag{1.1}$$

where f is assumed to be continuous on a subset of $\mathbb{R}^n$ and solutions of initial value problems associated with (1.1) exist, are unique and extend throughout a fixed subinterval of $\mathbb{R}$.

In Section 2, a monotonicity restriction on f ensures that the following $(n-1)$-point boundary value problems

$$\begin{aligned} &y^{(n)} = f(x, y, y', \dots, y^{(n-1)}) \\ &y(x_j) = y_j \quad (j = 1, 2, \dots, n-1), \quad y^{(i)}(x_2) = m \quad (i = 1, 2) \end{aligned} \tag{1.2$_i$}$$

$$\begin{aligned} &y^{(n)} = f(x, y, y', \dots, y^{(n-1)}) \\ &y(x_k) = y_k \quad (k = 2, \dots, n), \quad y^{(i)}(x_2) = m \quad (i = 1, 2) \end{aligned} \tag{1.3$_i$}$$

have at most one solution and with the added hypothesis that solutions exist to (1.2_i) and (1.3_i) $(i = 1, 2)$, a unique solution to the n-point boundary value problem

$$\begin{aligned} &y^{(n)} = f(x, y, y', \dots, y^{(n-1)}) \\ &y(x_\ell) = y_\ell \quad (\ell = 1, 2, \dots, n) \end{aligned} \tag{1.4}$$

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