Differential and Integral Equations, Volume 4, Number 2, March 1991, pp. 397–410.

# EXISTENCE AND UNIQUENESS THEOREMS FOR A FOURTH ORDER BOUNDARY VALUE PROBLEM OF STURM–LIOUVILLE TYPE†

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(Submitted by: Klaus Schmitt)

**Abstract.** This paper is devoted to obtaining natural existence and uniqueness theorems for the fourth-order boundary value problem $\frac{d^4u}{dx^4} + f(x, u(x), u'(x), u''(x)) = e(x)$, $0 < x < 1$, $u(0) = u(1) = 0$, $u'''(0) - hu''(0) = 0$, $u'''(1) + ku''(1) = 0$, $h \geq 0$, $k \geq 0$, $h + k > 0$, using degree theoretic methods for any given $e \in L^1[0,1]$. The function $f : [0,1] \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is not required to be bounded on $[0,1] \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}$ and satisfies conditions that are natural to the boundary value problem.

**1. Introduction.** Let $f : [0,1] \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ be a function satisfying Caratheodory's conditions, $e : [0,1] \to \mathbb{R}$ be a function in $L^1[0,1]$, and $h$, $k$ be real numbers such that $h \geq 0$, $k \geq 0$, $h+k > 0$. The purpose of this paper is to study the existence and uniqueness of solutions for the fourth-order boundary value problem

$$\frac{d^4u}{dx^4} + f(x, u, u', u'') = e(x), \quad 0 < x < 1, \tag{1.1}$$

$$u(0) = u(1) = 0, \tag{1.2}$$

$$u'''(0) - hu''(0) = 0, \quad u'''(1) + ku''(1) = 0. \tag{1.3}$$

Various fourth-order boundary value problems are used to model deformations of an elastic beam depending on how the beam is supported at the two endpoints [13]. They have been studied extensively in recent times by several authors (see, e.g., [1-3, 7-11, 14-15]). Seemingly motivated by the works of Bebernes-Gaines [4] and Corduneanu [6] on second-order generalized boundary value problems, Aftabizadeh

Received January 26, 1989.
†This work was supported in part by the Applied Mathematical Sciences subprogram of the Office of Energy Research, U.S. Department of Energy, under Contract W-31-109-Eng-38 under the Faculty Research Leave Program.
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AMS Subject Classifications: 34B10, 34B15, 34B25.