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ON THE OPTIMAL CONTROL PROBLEMS OF PARABOLIC EQUATIONS WITH AN INFINITE NUMBER OF VARIABLES AND WITH EQUALITY CONSTRAINTS

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Abstract. Optimal control problems of systems governed by parabolic equations with an infinite number of variables and with additional equality constraints are considered. The extremum principles as well as sufficient conditions of optimality are formulated by using certain extensions of the Dubovitskii-Milyutin method.

Introduction. The problem of optimal control of systems governed by parabolic or hyperbolic equations has been discussed in several papers starting with [10].

In [7] and [8] the Dubovitskii-Milyutin method has been applied to prove the extremum principles for such systems. These results have been extended to cases of optimal control of hyperbolic or parabolic systems with additional (operator or nonoperator) equality constraints in [13] and [14] by using extensions of the Dubovitskii-Milyutin method from [12] and [15]. All these results concerned optimal control problems of systems with a finite number of variables. In [1], [2] and [3] optimal control problems of a system governed by elliptic or hyperbolic operators with an infinite number of variables have been considered. Necessary conditions of optimality have been obtained by using the method of Lions (cf. [10]). In [9] results similar to [8] in the form of the extremum principle have been obtained for the case of parabolic equations with an infinite number of variables by using the Dubovitskii-Milyutin method from [5].

In the present paper, the above results for systems governed by parabolic equations are extended to the case of optimal control with additional equality constraints and with an infinite number of variables. By using the extension of the Dubovitskii-Milyutin method from [12] and [13] the extremum principle, as well as the sufficient condition of optimality, are proved for the optimal control problem of a parabolic equation with infinitely many variables and with the additional terminal constraint $y(x,T) = 0$. Then another optimal control problem of a parabolic equation with infinitely many variables but with nonoperator equality constraint $u(\cdot) \in U$ is considered and the extremum principle is proved for it by using the generalization of the Dubovitskii-Milyutin method from [15].

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