Differential and Integral Equations, Volume 4, Number 2, March 1991, pp. 305–329.

A CLASS OF INTEGRABLE HAMILTONIAN SYSTEMS INCLUDING SCATTERING OF PARTICLES ON THE LINE WITH REPULSIVE INTERACTIONS

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Abstract. The main purpose of this paper is to introduce a new class of Hamiltonian scattering systems of the cone potential type that can be integrated via the asymptotic velocity. For a large subclass, the asymptotic data of the trajectories define a global canonical diffeomorphism $\mathcal{A}$ that brings the system into the normal form $\dot{P} = 0$, $\dot{Q} = P$. The integrability theory applies for example to a system of n particles on the line interacting pairwise through rather general repulsive potentials. The inverse r-power potential for arbitrary $r > 0$ is included, the reduction to normal form being carried out for the exponents $r > 1$. In particular, the Calogero system is obtained for $r = 2$. The treatment covers also the nonperiodic Toda lattice. The cone potentials that we allow can undergo small perturbations in any arbitrary compact set without losing the integrability and the reduction to normal form.

1. Introduction.

An autonomous $2n$-dimensional Hamiltonian system is said to be integrable if there exist n smooth first integrals, independent and in involution (see e.g. [1] for details). The *scattering* systems provide natural constants of motion: the *asymptotic velocities*. Unfortunately, there is no obvious reason for them to be smooth functions of the initial data, and in fact they are sometimes not even continuous (see e.g. [7], §3). Some work has been done to single out classes of scattering-type systems for which rigorous proofs of smoothness (C^k, $2 \le k \le +\infty$) of the asymptotic data and of integrability could be carried out.

In [7] we investigated the *complete integrability* of Hamiltonian systems of the form

$$\dot{p} = -\nabla \mathcal{V}(q)\,, \quad \dot{q} = p\,, \quad p, q \in \mathbb{R}^n, \tag{1.1}$$

where $\mathcal{V}$ is a C^k ($3 \le k \le +\infty$) potential—defined on a domain that we can suppose for now to be simply $\mathbb{R}^n$—with the following basic properties:

CP_1 it is *bounded below* (say, $\mathcal{V} \ge 0$);

Received December 28, 1989.
This research was supported by the Ministero della Pubblica Istruzione and by the C.N.R.
AMS Subject Classifications: 34D05, 70H05, 58F07.