

**COMPARISON THEOREMS FOR FIRST ORDER
RETARDED FUNCTIONAL DIFFERENTIAL EQUATIONS
AND THEIR APPLICATIONS**

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Abstract. We obtain some comparison theorems for nonlinear retarded functional differential equations. We also establish some results about asymptotic behavior of all solutions to nonoscillatory equations. Oscillation criteria for nonlinear functional differential equations are given.

1. Introduction. Let $C = \{\phi : \phi : (-\infty, 0] \rightarrow \mathbf{R} \text{ is continuous, } \lim_{s \rightarrow -\infty} \phi(s) \text{ exists and is finite}\}$ and designate the norm of an element ϕ in C by $\|\phi\| = \sup_{s \leq 0} |\phi(s)|$. Clearly, C is a Banach space.

In this paper we shall first be interested in establishing comparison theorems for the retarded functional differential inequalities

$$x(t)\{x'(t) + f(t, x_t)\} \leq 0, \quad t \geq t_0, \quad (1)$$

and

$$y(t)\{y'(t) + f(t, y_t)\} \geq 0, \quad t \geq t_0, \quad (2)$$

where

$f : [t_0, \infty) \times C \times \mathbf{R}$ is continuous,

$x : (-\infty, A) \rightarrow \mathbf{R}$ is continuous, $t_0 < A \leq \infty$,

$x_t : (-\infty, 0] \rightarrow \mathbf{R}$, $t_0 \leq t < A$, $x_t(s) = x(t + s)$, $s \leq 0$.

(Clearly, $x : [t_0, A) \rightarrow C$ is continuous.)

Kwong and Patula [9] have obtained some comparison theorems for the retarded differential difference equations

$$x'(t) + \sum_{i=1}^n q_i(t)x(t - T_i(t)) = 0, \quad t \geq 0 \quad (3)$$

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