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MONOTONICITY IN TIME AT THE SINGLE POINT FOR THE SEMILINEAR HEAT EQUATION WITH SOURCE

VICTOR A. GALAKTIONOV
Keldysh Institute of Applied Mathematics, USSR Academy of Sciences
Miusskaya Sq., 4, 125047 Moscow, USSR

(Submitted by A.R. Aftabizadeh)

Abstract. We prove some results on monotonicity in time at the single spatial point $x = 0$ of the solutions of the semilinear heat equation $u_t = \Delta u + u^\beta$ in $\mathbb{R}^N \times (0, T)$, where $\beta > 1$ is a fixed constant. It is shown that under some hypotheses on the initial function any large solution $u = u(|x|, t)$ becomes monotone in time at the point $x = 0$. It is proved that under some hypotheses monotonicity of the solution at the single point does not depend on the behavior of the initial function in some large domain. The conditions of monotonicity are quite different for the three cases $\beta < \beta_*$, $\beta = \beta_*$ and $\beta > \beta_*$, where $\beta_* = (N + 2)/(N - 2)$ for $N > 2$ is the critical Sobolev exponent for the operator given in the right-hand side of the equation. The proofs are based on the methods of intersection's comparison (or lap number theory) with the infinite set of the stationary solutions of the equation considered.

1. Introduction, main results. In this paper we consider the Cauchy problem for the semilinear parabolic equation

$$u_t = \Delta u + u^\beta \quad \text{in } \mathbb{R}^N \times (0, T), \tag{1}$$

$$u(x, 0) = u_0(x) \quad \text{in } \mathbb{R}^N, \tag{2}$$

where $\beta > 1$ is a fixed constant. Equation (1) describes processes of the heat conduction and combustion in the media with constant heat conductivity coefficient and the source of heat energy $Q(u) = u^\beta$ depending on the temperature $u = u(x, t)$.

We assume initial function (2), which is the initial temperature of the nonlinear medium, satisfies the following conditions:

$$\begin{aligned} &u_0 = u_0(r) > 0 \quad \text{in } \mathbb{R}^N, \quad r = |x|; \quad M_1 = \sup u_0 < \infty; \\ &u_0 \in C(\mathbb{R}^N), \quad u_0'(0) = 0. \end{aligned} \tag{3}$$

It is well known that under these hypotheses the Cauchy problem (1), (2) has a unique positive classical solution in $\mathbb{R}^N \times (0, T)$ for some $T \leq \infty$ (see [3]). Here T is the maximal existence time. If $T < \infty$, then the solution blows-up in the finite time; i.e.,

$$\overline{\lim_{t \to T^-}} \, \|u(\cdot, t)\|_{L^\infty(\mathbb{R}^N)} = \infty. \tag{4}$$

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