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AN ABSTRACT APPROACH TO A CLASS OF NONLINEAR BOUNDARY VALUE PROBLEMS

C. De Coster, C. Fabry and P. Habets
Institut de Mathématique, Université Catholique de Louvain
B-1348 Louvain-la-Neuve, Belgium

(Submitted by A.R. Aftabizadeh)

1. Introduction. A large class of nonlinear boundary value problems can be written under some abstract form

$$Lu = N(u). \tag{1.1}$$

Throughout the paper, L and N will satisfy the following assumptions:

(H-1) Let U and V be real normed spaces. The operator $L : \operatorname{dom} L \subset U \to V$ is a linear, Fredholm operator of index zero and $N : U \to V$ an L-completely continuous function (we refer to J. Mawhin [10] for the definitions).

A basic tool to investigate equation (1.1) is the coincidence degree, upon which the following continuation theorem is based.

Theorem 1. (J. Mawhin [10]). *Let L and N satisfy* (H-1). *Assume there exist an open bounded set $\Omega \subset U$ and an L-completely continuous function $D : U \to V$ such that*

(i) $\quad \forall u \in \operatorname{dom} L \cap \partial\Omega,\ \forall \lambda \in]0,1[, \quad Lu \neq \lambda N(u) + (1-\lambda)D(u);$ (1.2)

(ii) $\quad D_L(L - D, \Omega) \neq 0.$

Then there exists at least one solution u of (1.1).

In the above statement, D_L denotes the coincidence degree. To apply this result, one has to choose a set Ω and an operator D. The choice will be suggested by the structure of the problem. Notice that, if D is linear and $L - D$ one-to-one, condition (ii) is satisfied, provided that $0 \in \Omega$, the degree being then equal to $+1$ or -1.

In this paper, we will consider mainly nonlinearities having a decomposition of the form

$$N(u) = G(u)u + R(u), \tag{1.3}$$

where, for fixed u, $G(u)$ is a linear, or at least positively homogeneous, operator and where $R(u) = o(\|u\|)$ as $\|u\| \to \infty$. Assuming that the operator $G(u) : U \to V$ belongs to some convex set $\mathcal{E}(U, V)$, it is natural to relate the nonlinear equation (1.1) to the family of linear, or positively homogeneous, problems

$$Lu = Su, \tag{1.4}$$

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