Differential and Integral Equations, Volume 4, Number 5, September 1991, pp. 1005–1022.

CONDITIONS FOR PARABOLICITY OF SECOND ORDER ABSTRACT DIFFERENTIAL EQUATIONS*

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(Submitted by J.L. Bona)

Abstract. We study conditions ensuring that the differential equation $u'' + Bu' + Au = 0$ in a Banach space be parabolic. The model case when B is a scalar multiple of a fractional power of A is considered first: we obtain necessary and sufficient conditions, generalizing the Hilbert space results in [4] and [8]. Subsequently we prove some sufficient conditions, when B is stronger than $A^{1/2}$ which are of a different nature from those in [4]. Examples of applications to initial boundary value problems for partial differential equations are also given.

1. Introduction. In recent years many papers were published on second order differential equations in Banach spaces (see [7], §2.5(c), where many references up to 1980 can be found, and, among the latest papers, [2], [5], [11], [15], [17]). Most of them deal with either applications of cosine function theory or equations of general type

$$u''(t) + Bu'(t) + Au(t) = f(t), \quad t \in \mathbb{R}^+, \tag{1.1}$$

without assumptions of parabolicity (see §2 for a precise definition).

Let us now examine some papers which are more connected with ours. In [10], V. Mel'nikova and A.I. Filinkov give a sort of classification of equations such as (1.1), with A and B commuting, based upon subordination relations (in the sense of [9], Ch. I, §7) of one coefficient on the other one, but most of their results are concerned with non-parabolic equations. In [22], S.Ya. Yakubov considers an equation similar to (1.1) and, by reducing it to a first order system in a suitable way, proves an existence and uniqueness result, under hypotheses similar to those in our Theorem 4.1, which imply parabolicity. In the papers [12], [13] by the second author and [18], [19] by H. Tanabe, evolution operators for parabolic, n-th order equations with time dependent coefficients are exhibited, so allowing to prove existence and uniqueness results of the solutions of the Cauchy problem. In [4] , S. Chen and R. Triggiani, assuming that the operators A, B in (1.1) are positive and selfadjoint in a Hilbert space, give conditions in order that the closure of the operator, obtained by transforming equation (1.1) into a first order system in the usual way, generates an analytic semigroup in suitable product spaces. Their hypotheses require that $\mathcal{D}(B^{1/2}) = \mathcal{D}(A^{\alpha/2})$, where $\alpha \in [\frac{1}{2}, 1]$, and they show by a counterexample that, if

Received March 1990, in revised July 1990.
*Partially Supported by M.U.R.S.T., Italy (Fondi 40%) and by Universitá di Bologna (Fondi 60%).
AMS Subject Classifications: 34G10, 35K30.