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INERTIAL MANIFOLDS AND THE SLOW MANIFOLDS IN METEOROLOGY

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Abstract. Numerous articles have been devoted to the construction of inertial manifolds for dissipative evolution equations. Most of them are limited to the case where the linear operator in the equation is self-adjoint and the inertial manifold is built as the graph over an eigenspace of that operator.

Our aim in this article is to address the more general case where the linear operator in the equation is non self-adjoint (but is however a compact perturbation of such operator). And we build the inertial manifold as the graph over a root space of that operator. The construction of the inertial manifold uses the Lyapunov-Perron method as in C. Foias, G. Sell and R. Temam [1]; however, we had to derive here some delicate spectral properties of non self-adjoint linear operators that we did not find available in the literature.

One of the motivations of this work is obviously to extend the concept of inertial manifolds and obtain *new classes of inertial manifolds.* The second and most important motivation is to recover the concept of *slow manifold* which is broadly used in weather forecast in relation with data assimilation (see e.g., L. Bengtsson, M. Ghil and E. Kallen [1], J.J. Tribia [1]). By linearizing the evolution equation (of Navier-Stokes type) around a stationary solution, we obtain an evolution equation with a non self-adjoint linear part. By applying the above results to this equation we obtain an inertial manifold that satisfies all the properties of the slow manifold that were conjectured in the meteorology literature: the slow manifold contains the origin; it is tangent at the origin to the root space on which it is built; and it attracts all trajectories at an exponential rate.

Introduction. Consider the equation involving the unknown u in a Hilbert space H :

$$\frac{du}{dt} + Au = R(u), \quad u(0) = u_0, \tag{0.1}$$

where A is an unbounded selfadjoint linear operator and R a nonlinear function. Under suitable assumptions, we can associate to the system (0.1) a semigroup $\{S(t)\}_{t\geq 0}$. We recall that a set $\mathcal{M}$ is an inertial manifold for (0.1) or the semigroup $\{S(t)\}_{t\geq 0}$ if $\mathcal{M}$ is a finite dimensional lipschitz manifold positively invariant that attracts all the orbits of (0.1) with an exponential speed. The interest of such a set is that it allows the reduction of (0.1) to a finite dimensional system that enjoys the same asymptotic properties. In particular, due to its positive invariance $\mathcal{M}$ contains transient trajectory as well as the attractor of (0.1).

Numerous articles have been devoted to the construction of inertial manifolds for infinite dimensional dynamical systems generated by partial differential equations

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