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A SINGULAR BOUNDARY VALUE PROBLEM ARISING FROM NEAR-IGNITION ANALYSIS OF FLAME STRUCTURE

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Abstract. The boundary value problem

$$\frac{d^2y}{dx^2} + Qf(x)\exp(y) = 0, \quad 0 < x < 1 \tag{1}$$

$$y(0) = y(1) = 0 \tag{2}$$

is studied. Here Q is a given non-negative constant; $f(x)$ is in $C^1(0,1]$ and $f(x) > 0$ for x belonging to $(0,1)$. With a controlled blow-up rate of f near $x = 0$, it is shown that there exists a $Q_0 > 0$ such that for $0 \leq Q < Q_0$, a solution in $C[0,1] \cap C^2(0,1]$ for the boundary value problem exists, while for $Q > Q_0$, there is no solution.

1. Introduction. Study of the structure of diffusion flame near ignition leads to equations (1) and (2) [1–3]. Specifically, $f(x) = x^{L_o-2}(1 - x^{L_f})$ in such a circumstance, where L_0, L_f are the oxidant and fuel Lewis numbers, respectively. In this paper, we allow for a more general situation for f. It is assumed that it satisfies the following:

(i) $f(x)$ is in $C^1(0,1]$,
(ii) $f(x) > 0$ for x belonging to $(0,1)$,
(iii) $f(x)$ can be singular at $x = 0$, but is at most $O(\frac{1}{x^{2-\delta}})$ as $x \to 0^+$ for some $\delta > 0$.

It can then be shown that there exists a $Q_0 > 0$ such that for $0 \leq Q < Q_0$, a solution of (1) and (2) exists in $C[0,1] \cap C^2(0,1]$, whereas for $Q > Q_0$, there is no solution. The strategy of the proof involves a shooting argument and the construction of an upper and a lower solution for the equation. The result is consistent with the numerical result, which is summarized in Figures 1 and 2.

2. Arguments in the proof. We will establish the result through the following lemmas.

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