Differential and Integral Equations, Volume 4, Number 4, July 1991, pp. 835–850.

EXISTENCE AND UNIQUENESS FOR THE POPULATION DYNAMICS WITH NONLINEAR DIFFUSION

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Abstract. In this paper some existence and uniqueness results in $H^{-1}(\Omega)$ and $L^1(\Omega)$ for population dynamics with nonlinear diffusion are proved. The same questions are studied when the initial datum is a measure.

1. Introduction. The first part of this paper is concerned with the existence and uniqueness in $H^{-1}(\Omega)$ and $L^1(\Omega)$ of the solutions for the following population dynamics with nonlinear diffusion:

$$\begin{cases} u_t + u_a + \mu(a)u - \Delta_x \beta(u) \ni f \quad \text{on } (0,A)\times\Omega\times(0,T) \\ \beta(u(a,x,t)) \ni 0 \quad \text{on } (0,A)\times\partial\Omega\times(0,T) \\ u(0,x,t) = \displaystyle\int_0^A b(a)u(a,x,t)\,da \quad \text{on } \Omega\times(0,T) \\ u(a,x,0) = u_0(a,x) \quad \text{on } (0,A)\times\Omega, \end{cases} \tag{1.1}$$

where Ω is a bounded and open subset of the Euclidean space $\mathbb{R}^N$ $(N \geq 1)$ with sufficiently smooth boundary. The role of nonlinear diffusion in population dynamics is emphasized in [2]. The significance of the terms in (1.1) can be found in [1, 14].

Next, when $m > (N-2)/N$ we prove the existence of the solution of the following problem:

$$\begin{cases} u_t + u_a + \mu(a)u - \Delta_x(|u|^{m-1}u) = 0 \quad \text{on } (0,A)\times\Omega\times(0,T) \\ u(a,x,t) = 0 \quad \text{on } (0,A)\times\partial\Omega\times(0,T) \\ u(0,x,t) = \displaystyle\int_0^a b(a)u\,da \quad \text{on } \Omega\times(0,T) \\ u(a,x,0) = u_0(a)\otimes\nu \quad \text{on } (0,A)\times\Omega, \end{cases} \tag{1.2}$$

where $u_0 \in L^1(0,A)$ and $\nu \in M^+(\Omega)$. Here $M^+(\Omega)$ is the set of all nonnegative Radon measures. If $m \geq 1$ we also prove the uniqueness of the solution. When

Received November 1989.
AMS Subject Classifications: 35D99, 35K65, 92A15.