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ASYMPTOTIC STABILITY FOR NONLINEAR DEGENERATE PARABOLIC EQUATIONS WITH NEUMANN BOUNDARY CONDITIONS

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Abstract. In this paper we study a quasilinear degenerate parabolic equation with Neumann boundary condition of the form $u_t - \Delta\beta(u) \ni f(t,x)$ in $\mathbb{R}\times\Omega$, $\partial_n\beta(u) \ni h(t,x)$ on $\mathbb{R}\times\partial\Omega$, where Ω is a bounded domain in $\mathbb{R}^N$ with smooth boundary $\partial\Omega$, β is a given maximal monotone graph in $\mathbb{R}\times\mathbb{R}$ and f, g are given functions. We shall show the existence of a periodic solution in time and its stability as $t \to +\infty$.

1. Introduction.

In this paper we study the following quasilinear parabolic equation:

$$u_t - \Delta\tilde{\beta} = f, \quad \tilde{\beta} \in \beta(u), \quad \text{in } \mathbb{R}\times\Omega, \tag{1.1}$$

$$\partial_n\tilde{\beta} = h \quad \text{on } \mathbb{R}\times\Gamma, \tag{1.2}$$

where Ω is a bounded domain in $\mathbb{R}^N$ $(N \ge 1)$ with sufficiently smooth boundary $\Gamma = \partial\Omega$; β is a given maximal monotone graph in $\mathbb{R}\times\mathbb{R}$; f and h are given functions on $\mathbb{R}\times\Omega$ and on $\mathbb{R}\times\Gamma$, respectively; the function $u = u(t,x)$, with $\tilde{\beta} = \tilde{\beta}(t,x) \in \beta(u)$, is the unknown; ∂_n denotes the outward normal derivative on Γ.

Equation (1.1) represents mathematical models of some physical problems, and there are three interesting cases (a), (b) and (c) of β mentioned below:

(a) β is Lipschitz continuous on $\mathbb{R}$ with linear growth at $\pm\infty$;
(b) β^{-1} is Lipschitz continuous on $\mathbb{R}$;
(c) the domain $D(\beta)$ of β is bounded and not a singleton in $\mathbb{R}$; i.e., $\overline{D(\beta)} = [r_*, r^*]$ for some $-\infty < r_* < r^* < +\infty$.

For instance, in the case (a) equation (1.1) includes the enthalpy formulation of Stefan problem (cf. [4, 9, 13, 19]), and in the case (b) or (c) it has been considered as a mathematical modeling of filtration in porous media and of flow in Hele-Shaw cells (cf. [1-3, 5-8, 12, 16, 17]).

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