Differential and Integral Equations, Volume 4, Number 4, July 1991, pp. 719–730.

NONRESONANCE CONDITIONS FOR THE EXISTENCE, UNIQUENESS, AND STABILITY OF PERIODIC SOLUTIONS OF DIFFERENTIAL EQUATIONS WITH A SYMMETRIC NONLINEARITIES

A.C. LAZER†
Department of Mathematics, University of Miami, Coral Gables, FL. 33124

P.J. MCKENNA†
Department of Mathematics, University of Connecticut, Storrs, CT 06268

(Submitted by: A.R. Aftabizadeh)

Abstract. We consider the differential equation $u'' + ku' + g(u) = p(t) + A\sin(mt + c)$, where $g'(\xi) \to b$ as $\xi \to \infty$, $g'(\xi) \to a$ as $\xi \to -\infty$, $k > 0$, p is continuous and 2π-periodic, and $m \geq 1$ is an integer. Conditions on a, b, k and m are given which ensure that for A sufficiently large and positive, there is a unique, asymptotically stable, 2π-periodic solution.

1. Introduction. The purpose of this paper is to extend the main result of [5]. In [5] we considered the differential equation

$$u''(t) + ku'(t) + bu^+(t) - au^-(t) = p(t) + A\sin(mt + c),$$

where $A, a, b,$ and k are positive constants with $a < b$, $m \geq 1$ is an integer, c is constant, and p is continuous and $2\pi-$periodic. Motivated by a numerical study carried out in [2], it was shown in [5] that if

$$\frac{(2m-1)^2}{4} < a \leq b < \frac{(2m+1)^2}{4},$$

and $k > 0$ is arbitrary, then for A sufficiently large, there exists a unique 2π-periodic solution of the above differential equation and it is locally asymptotically stable in the sense of Liapunov.

Recently, in [6], we proved that if $g(t, \xi)$ is continuous and 2π-periodic in t and has a continuous partial derivative with respect to the second variable, $a \leq D_2 g(t, \xi) \leq b$ for all $(t, \xi) \in \mathbb{R}^2$, where a and b are positive constants, $k > 0$ is constant, and there

Received July 1990.
†Partially supported by NSF Grant DMS-8722593.
†Partially supported by NSF Grant DMS-8722532.
AMS Subject Classifications: 34C25.