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MAXIMIZATION ON CLASSES OF FUNCTIONS WITH FIXED REARRANGEMENT

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Abstract. In this paper we deal with two maximization problems in classes of functions with fixed rearrangement: the maximization of an energy type functional and the rearrangement of the gradient. In the first one we give two counterexamples to the uniqueness of the maximum, while in the second one we study a case in which the shape of a maximizer can be found.

1. Introduction. Let Ω be an open bounded set in $\mathbb{R}^n$ and let $f \in L^p_+(\Omega)$. We denote the distribution function of f by

$$\mu_f(t) = \left|\{x \in \Omega : f(x) > t\}\right|$$

(for any set $A \subset \mathbb{R}^n$ we denote by $|A|$ its n-dimensional Lebesgue measure) and the decreasing rearrangement of f by

$$f^*(s) = \sup\{t \geq 0 : \mu_f(t) > s\}.$$

More generally, if $f, g \geq 0$ are measurable functions, we say that f is a rearrangement of g, $f \sim g$, if f and g have the same distribution function. For more details about properties of rearrangements see for example [7], [16], [23], [26].

For any fixed $f_0 \in L^p_+(\Omega)$, let

$$C(f_0) = \{f \in L^p_+(\Omega) : f \sim f_0\}$$

be the set of the rearrangements of f_0 in Ω. This paper is related to the problem of maximizing some functionals on the set $C(f_0)$. A standard way to approach this problem can be sketched as follows (see e.g. [2]). Let $K(f_0)$ be the set of all weak limits in $L^p(\Omega)$ of sequences in $C(f_0)$. By a useful characterization of $K(f_0)$ (see [29], [25]), it can be proved ([28], [2]) that $K(f_0)$ is a closed, weakly compact convex set in $L^p(\Omega)$ and $C(f_0)$ is the set of its extremal points (a point x is said to be extremal for a set A if it is not possible to find $x_1, x_2 \in A$, $x_1 \not\equiv x_2$, such that $x = (x_1 + x_2)/2$). Because of that, if we want to maximize a weakly continuous convex functional Φ on $C(f_0)$, we can say that at least one maximum

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