Differential and Integral Equations, Volume 6, Number 1, January 1993, pp. 131–153.

MULTIDIMENSIONAL TRAVELING WAVEFRONTS IN A MODEL FROM COMBUSTION THEORY AND IN RELATED PROBLEMS†

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(Submitted by: P.L. Lions)

Abstract. The paper deals with the flame propagation model $\partial u/\partial t = \Delta u + \beta(y)\partial u/\partial x + f(u)$ in $\Sigma = \{(x, y) \in \mathbb{R} \times \Omega\}$, $\partial u/\partial \nu = 0$ at $\partial\Sigma$, $0 = u(-\infty, \cdot) \leq u(x, \cdot) \leq u(\infty, \cdot) = 1$ in $\overline{\Omega}$ for all $x \in \mathbb{R}$. Here $\Omega \subset \mathbb{R}^{n-1}$ is a bounded domain with a sufficiently smooth boundary, ν is the unit outward normal to $\partial\Sigma$, $\beta : \Omega \to \mathbb{R}$ and $f : [0, 1] \to \mathbb{R}$ are sufficiently smooth and $f(0) = f(1) = 0$. Existence results concerning traveling wavefronts are given for that model and for related problems of practical interest.

1. Introduction. This paper deals with the existence of traveling wavefronts of the parabolic problem posed by

$$\partial u/\partial t = \Delta u + \beta(y)u_x + f(u) \quad \text{in} \quad t \geq 0, \quad (x, y) \in \mathbb{R} \times \Omega, \tag{1.1}$$

with appropriate boundary conditions at $\mathbb{R} \times \partial\Omega$. Here $\Omega \subset \mathbb{R}^{n-1}$ $(n \geq 2)$ is a bounded domain, with a $C^{2,\alpha}$ boundary for some $\alpha > 0$ if $n > 2$, and the spatial coordinates are written as (x, y), where $x = x_1$ and $y = (x_2, \ldots, x_n)$; Δ is the Laplacian operator and $\beta : \overline{\Omega} \to \mathbb{R}$ and $f : \mathbb{R} \to \mathbb{R}$ are appropriately smooth.

After a suitable change of variables, the wavefronts of (1.1) are solutions of

$$\Delta u + (c + \beta(y))u_x + f(u) = 0 \quad \text{in} \quad \mathbb{R} \times \Omega, \tag{1.2}$$

$$v_- = u(-\infty, \cdot) \leq u(x, \cdot) \leq u(\infty, \cdot) = v_+ \quad \text{in} \quad \Omega \quad \text{for all } x \in \mathbb{R}, \tag{1.3}$$

where the wave speed c is to be determined as a part of the solution. We shall say that the wavefront connects the rest states v_- and v_+.

The problem (1.2–1.3) with Neumann boundary conditions

$$\partial u/\partial \nu = 0 \quad \text{at} \quad \mathbb{R} \times \partial\Omega \tag{1.4}$$

has been recently considered in the literature. In particular, the case in which the nonlinearity f is such that $f(u) = 0$ if $0 \leq u \leq \theta$, $f(u) > 0$ if $\theta < u < 1$, $f(1) = 0$ and $f'(1) \neq 0$ was suggested by a problem from combustion theory and analyzed

Received September 1991.
†The research was partially supported by CICYT and DGICYT, under Grants ESP-90-0187 and PB90-0271.
AMS Subject Classification: 35J65, 35K57, 35K60, 80A32.