Differential and Integral Equations, Volume 4, Number 6, November 1991, pp. 1303–1310.

ON ASYMPTOTIC STABILITY FOR LINEAR DELAY EQUATIONS

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1. Introduction. For a linear scalar delay equation

$$\dot{x}(t) + ax(t) + bx(t-r) = 0,$$

the stability of the zero solution can be determined by whether all roots of the characteristic equation

$$\lambda + a + be^{-\lambda r} = 0 \tag{1.1}$$

lie in the left half plane. And it is well known [1] that all roots of (1.1) lie in the left half plane if either

$$a > -b \geq -\frac{1}{r}$$

or

$$rb = \frac{\theta}{\sin\theta} \quad \text{and} \quad a > -b\cos\theta \quad \text{for some } \theta \in (0,\pi).$$

In this paper, we extend this result to a more general equation of the form

$$\lambda + a + \int_0^r d\eta(s)e^{-\lambda s} = 0, \tag{1.2}$$

where η is a function of bounded variation on $[0,r]$ and $\int_0^{0^+} d\eta(s) = 0$, and then apply it to discuss the stability of some classes of delay equations, including a partial delay-differential equation studied by Green and Stech [2].

2. A main theorem. In this section we shall establish a theorem concerning the location of the roots of (1.2).

Lemma 2.1. *Let $\theta \in (0,\pi)$, then $\theta\cos\theta/\sin\theta < 1$.*

Proof: Since

$$\frac{d}{d\theta}(\cos\theta\sin\theta - \theta) = -2\sin^2\theta < 0, \qquad \theta \in (0,\pi),$$

and $\cos\theta\sin\theta - \theta|_{\theta=0} = 0$, it follows that $\cos\theta\sin\theta - \theta < 0$, for $\theta \in (0,\pi)$. Hence

$$\frac{d}{d\theta}\left(\frac{\theta\cos\theta}{\sin\theta}\right) = \frac{\cos\theta\sin\theta - \theta}{\sin^2\theta} < 0, \qquad \theta \in (0,\pi).$$

Note that $\theta\cos\theta\sin\theta \to 1$ as $\theta \to 0^+$, therefore, $\frac{\theta\cos\theta}{\sin\theta} < 1$, $\theta \in (0,\pi)$.

Received for publication July 1989.
†This paper constitutes part of the author's Ph.D. dissertation at the Claremont Graduate School.
AMS Subject Classifications: 34.