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REMARKS ON SEMILINEAR ELLIPTIC PROBLEMS AT RESONANCE*

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Abstract. In this paper we study the solvability of nonlinear elliptic equations of resonance type in which the nonlinearity is bounded but does not satisfy the Landesman-Lazer condition.

1. Introduction. Let $\Omega \subset \mathbb{R}^N$ be a smooth bounded domain and $g : \mathbb{R} \to \mathbb{R}$ a bounded continuous function. We are concerned with the question of existence of solutions for the problem

$$-\Delta u - \lambda_k u + g(u) = h(x) \quad \text{in } \Omega, \quad u \in H_0^1(\Omega) \tag{1}$$

where λ_k denotes an eigenvalue with multiplicity $p \geq 1$ of the eigenvalue problem

$$-\Delta u = \lambda u \quad \text{in } \Omega, \quad u \in H_0^1(\Omega) \tag{2}$$

and h is an L^2-function. We recall that the eigenvalues of (2) form an infinite sequence

$$\lambda_1 < \lambda_2 \leq \ldots \leq \lambda_k, \ldots ,$$

each one occurring in the sequence as often as its multiplicity, and the corresponding sequence of eigenfunctions

$$\varphi_1, \varphi_2, \cdots , \varphi_k, \cdots$$

is a complete orthonormal system for $L^2(\Omega)$. Moreover, λ_1 is positive and simple and we may choose $\varphi_1 > 0$ in Ω. We set

$$\lambda_{k-p} < \lambda_{k-p+1} = \ldots = \lambda_k < \lambda_{k+1}$$

with $\lambda_0 \equiv -\infty$ and the nullspace relative to λ_k is

$$N_k = \text{ span}\{\varphi_{k-p+1}, \cdots , \varphi_k\}.$$

In the famous Landesman-Lazer Theorem [1] the function g is supposed to have finite limits at infinity, say

$$g(u) \to g_\pm \quad \text{as} \quad u \to \pm\infty$$

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