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NECESSARY CONDITIONS FOR THE CONVERGENCE AND SUMMABILITY OF EIGENFUNCTION EXPANSIONS IN THE COMPLEX DOMAIN

W. EBERHARD
Universität-GH-Duisburg, FB 11-Mathematik, Lotharstr. 65, D-4100, Duisburg, FRG

G. FREILING
Lehrstuhl II für Mathematik, RWTH Aachen, Templergraben 55, D-5100 Aachen, FRG

(Submitted by: A.R. Aftabizadeh)

1. Introduction. We consider non-selfadjoint eigenvalue problems

$$l(y,\lambda) = \sum_{j=1}^{n} \Big(\sum_{k=0}^{j} p_{kj}(z) y^{(k)} \Big) \lambda^{n-j} + \lambda^n y = 0 \tag{1.1}$$

$$U_\nu(y) = 0, \quad 1 \le \nu \le n, \tag{1.2}$$

with two-point boundary conditions.

$\lambda_0 \in \mathbb{C}$ is an *eigenvalue* of (1.1), (1.2) if there is a nontrivial function φ satisfying $l(\varphi, \lambda_0) = 0$ and $U_\nu(\varphi) = 0$, $1 \le \nu \le n$; φ is called *eigenfunction* corresponding to λ_0.

If (1.1), (1.2) has a countable set $\sigma = \{\lambda_k : k \in \mathcal{N}\}$ of eigenvalues and if for $\delta > 0$ there are constants p and $c(\delta) > 0$ such that Green's function G of (1.1), (1.2) is defined for $\lambda \notin \sigma$ and satisfies

$$|G(x,\xi,\lambda)| \le c(\delta)|\lambda|^p \quad \text{for} \quad dist(\lambda,\sigma) \ge \delta,$$

then (1.1), (1.2) is called *(almost-) regular* (or *Stone-regular*); otherwise (1.1), (1.2) is called *irregular.*

For the investigation of regular or selfadjoint eigenvalue problems there exist various powerful methods and numerous results; especially it is possible in this case to expand every sufficiently smooth function satisfying adequate boundary conditions into a uniformly convergent series in e.a.f.'s (eigen- and associated functions) of (1.1), (1.2) (cf. [16], [12] and [8] for details and further references).

If (1.1), (1.2) is irregular then $G(x,\xi,\lambda)$ can grow exponentially for $|\lambda| \to \infty$. Irregular eigenvalue problems of the form (1.1), (1.2) and of more special type have been investigated by Ward [18], Eberhard [2], Hromov [10], [11] and Freiling [5], [6], [7]. These authors proved that only a very restricted class of functions can

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