Differential and Integral Equations, Volume 4, Number 6, November 1991, pp. 1121–1132.

ON THE EXACT CONTROLLABILITY OF KIRCHOFF PLATES

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Abstract. J. Lagnese and J.-L. Lions proved the exact controllability of some Kirchoff plate models by applying simultaneously two control functions. We prove here that in fact one control function is sufficient. The proof is based on an abstract norm strengthening method which may be useful in other situations as well.

1. Introduction. Let Ω be a non-empty bounded domain in $\mathbb{R}^n$ $(n \geq 1)$ having a boundary Γ of class C^4. Fix a point $x^0 \in \mathbb{R}^n$ arbitrarily and introduce the notations

$$\begin{aligned} m(x) &= x - x^0, \quad x \in \mathbb{R}^n, \\ R &= \sup\{|m(x)| : x \in \Omega\}, \\ \Gamma_+ &= \{x \in \Gamma : m(x) \cdot \nu(x) > 0\}, \quad \Gamma_- = \Gamma \setminus \Gamma_+, \end{aligned}$$

where $\nu(x)$ denotes the outward unit normal vector to Γ and $\cdot$ denotes the scalar product of $\mathbb{R}^n$.

Fix two (strictly) positive numbers h and T and consider the system:

$$y'' - h^2\Delta y'' + \Delta^2 y = 0 \quad \text{in } \Omega \times (0,T) \tag{1.1}$$

$$y(T) = y'(T) = 0 \quad \text{in } \Omega \tag{1.2}$$

$$y = \Delta y = 0 \quad \text{on } \Gamma_- \times (0,T) \tag{1.3}$$

$$y = 0 \text{ and } \Delta y = v \quad \text{on } \Gamma_+ \times (0,T). \tag{1.4}$$

For $n = 2$ this system provides a well-known plate model; cf. [3].

We shall prove that for every $v \in L^2(0,T;L^2(\Gamma_+))$ this system has a unique solution satisfying

$$y \in C([0,T]; H^2(\Omega) \cap H_0^1(\Omega)) \text{ and } y'(0) \in H_0^1(\Omega). \tag{1.5}$$

In particular, all exactly null-controllable states $(y(0), y'(0))$, where the admissible controls v run over $L^2(0,T;L^2(\Gamma_+))$, belong to $(H^2(\Omega) \cap H_0^1(\Omega)) \times H_0^1(\Omega)$.

We shall prove that all initial states from $(H^2(\Omega) \cap H_0^1(\Omega)) \times H_0^1(\Omega)$ are exactly null-controllable if T is sufficiently large.

Received December, 1990.
AMS Subject Classifications: 93B05, 31B30.