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A CONTRACTION SEMIGROUP GENERATED BY A PSEUDO-DIFFERENTIAL OPERATOR*

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Abstract. Let m be a positive number and $q : \mathbb{R}^n \to \mathbb{C}$ a measurable function. We give conditions on the function q to ensure that the pseudo-differential operator $-(-\Delta + m^2)^{\frac{1}{2}} + q$ is the infinitesimal generator of a one-parameter strongly continuous semigroup of contractions on $L^p(\mathbb{R}^n)$, $p > n$.

1. Introduction. We begin by recalling the definition and some important properties of dissipative operators in Lumer and Phillips [3]. To this end, we use the notion of a semi-inner-product in Lumer [2]. A closed linear operator A densely defined on a complex Banach space X is said to be dissipative if there is a semi-inner-product $(\cdot,\cdot)$ on X compatible with the norm $\|\cdot\|$ of X such that $\mathrm{Re}\,(Ax,x) \leq 0$ for all x in the domain $\mathcal{D}(A)$ of A. The property of being dissipative depends on the choice of the semi-inner-product in general. An important result in the theory of dissipative operators especially pertinent to the present paper is given in the following theorem.

Theorem 1.1. *Let A be a closed linear operator densely defined on a complex Banach space X. Then A is the infinitesimal generator of a one-parameter strongly continuous semigroup of contractions on X if and only if A is dissipative and there exists a positive number λ such that the range of $A - \lambda I$ is equal to X.*

Since the property of being the infinitesimal generator of a one-parameter strongly continuous semigroup of contractions on X is independent of the semi-inner-product, it follows that if A is a closed linear operator densely defined on X such that the range of $A - \lambda I$ is equal to X for some $\lambda > 0$, then dissipativity of A with respect to one semi-inner-product on X compatible with the norm of X implies dissipativity of A with respect to all semi-inner-products on X compatible with the norm of X.

Let B be another linear operator defined on the same Banach space X with dense domain $\mathcal{D}(B)$. Suppose that $\mathcal{D}(B)$ contains $\mathcal{D}(A)$ and $B : \mathcal{D}(A) \to X$ is a compact operator if we equip $\mathcal{D}(A)$ with the graph norm of A. Then we say that B is A-compact. The following theorem tells us when $A + B$ is the infinitesimal generator of a one-parameter strongly continuous semigroup of contractions on X.

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