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REGULARITY OF POINTWISE BOUNDARY CONTROL SYSTEMS

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Abstract. We will in these notes address some problems arising in "real-life" control application, namely problems concerning distributional control inputs on the boundary of the spatial domain. We extend the classical variational approach and give easily checkable sufficient conditions for the solutions to be continuous functions of time with values in $L^2(\Omega)$.

1. Introduction. We will consider the systems

$$\begin{aligned} &\partial_t u + Au = 0 && \text{in } \Omega, && \text{for } 0 < t < T,\\ &\gamma u = \delta_{a'}(x) f(t) && \text{on } \Gamma, && \text{for } 0 < t < T,\\ &u = 0 && \text{in } \Omega, && \text{at } t = 0, \end{aligned} \tag{1.1}$$

and

$$\begin{aligned} &\partial_t^2 u + Au = 0 && \text{in } \Omega, && \text{for } 0 < t < T,\\ &\gamma u = \delta_{a'} f(x) && \text{on } \Gamma, && \text{for } 0 < t < T,\\ &u = 0 && \text{in } \Omega, && \text{at } t = 0,\\ &\partial_t u = 0 && \text{in } \Omega, && \text{at } t = 0. \end{aligned} \tag{1.2}$$

Here $\delta_{a'}$ is the point measure in $a' \in \Gamma$, the (smooth) boundary of the bounded spatial domain Ω, and f is a function of time only. A is a uniformly strongly elliptic, formally self-adjoint partial differential operator of order $2m$ and γ is the Dirichlet trace operator

$$\gamma = \{\gamma_j\}_{0 \le j < m}, \tag{1.3}$$

where

$$\gamma_j u = \Big(-i \frac{\partial}{\partial n}\Big)^j u|_\Gamma \tag{1.4}$$

and n is the normal. We have also the Neumann trace operator

$$v = \{\gamma_j\}_{m \le j < 2m} \tag{1.5}$$

appearing in *Greens formula* for A:

$$(Au|w) - (u|Aw) = (\mathcal{A}^{01} vu + \mathcal{A}^{00}\gamma u|\gamma w)_\Gamma + (\mathcal{A}^{10}\gamma u|vw)_\Gamma, \tag{1.6}$$

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